

Numerical Measures of Fairness

Efficiency Gap

Consider the two district maps below, depicting two different districting options for a state that will get ten congressional seats (like Washington):

◆	○	○	○	○
◆	◆	◆	◆	◆
○	◆	◆	○	○
○	◆	◆	○	◆
◆	◆	◆	○	◆
◆	◆	◆	○	◆
◆	○	○	○	◆
◆	◆	◆	○	○

District Map Y

◆	○	○	○	○
◆	◆	◆	◆	◆
○	◆	◆	○	○
○	◆	◆	○	◆
◆	◆	◆	○	◆
◆	◆	◆	○	◆
◆	○	○	○	◆
◆	◆	◆	○	○

District Map Z

1. Compare the two district maps in terms of the following:
 - a. Number of districts
 - b. Average area per district (assume each rectangle is the same size)
 - c. Average population per district¹
 - d. Number of congressional seats won by each party

2. Which district map is fairer? Why do you say that?

¹ In *Wesberry v. Saunders* (1963), the Supreme Court ruled that the Equal Protection Clause of the 14th Amendment required roughly equal population sizes per district within a state. The congressional districts in question were in Georgia.

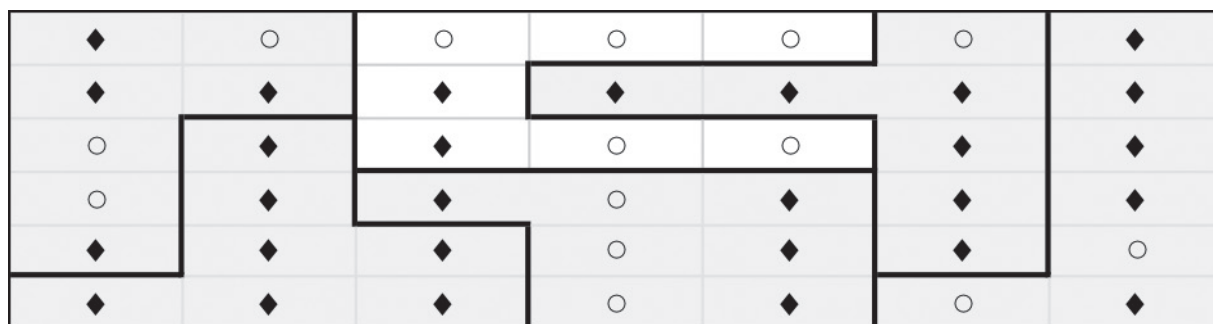
3. Given your answer to (2), do any of your answers to (1) surprise you? Why or why not?

4. In *Whitford v. Gill* (2016), a federal court declared a legislative map invalid because it evidenced gerrymandering. A significant motivator for the court's finding was the **efficiency gap** (EG). The EG is a ratio where values closer to 0 are seen as evidencing a fair districting plan and values close to +1 are seen as unfair toward one party. EG is defined as

$$EG = \frac{W_A - W_B}{T}$$

Where W_A and W_B are the total wasted votes for Party A and Party B, respectively, and T is the total number of votes cast.

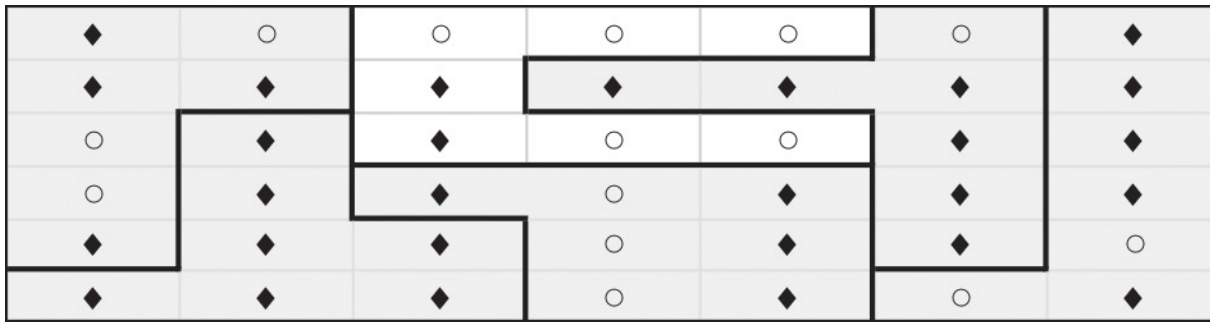
How can a vote be wasted? In order to win a district, Party A needs to get ____ percent of the vote. Consider the district map below. For your convenience, districts won by Party B are shaded.



District Map Q

In each district there are seven votes. Thus, in order to win the district, a party needs to win ____ votes. All votes in excess of this are termed wasted votes for the winning party. Why?

In the event that a party loses a district, all of its votes in that district are considered wasted votes. Why?



District Map Q (reproduced from previous page)

5. Find W_A , the number of wasted votes for Party A. Express your answer as a sum of the wasted votes in each district.

6. Find W_B , the number of wasted votes for Party B. Express your answer as a sum of the wasted votes in each district.

7. Find T , the total number of votes cast in this election.

8. Calculate the EG. Nicholas Stephanopoulos and Eric McGhee, who created the idea of the EG, proposed that the cutoff for unfair districting was $|EG| \geq 0.07$. Is District Map Q fair? Explain your answer, referencing your EG and the 0.07 threshold just mentioned.

Polsby-Popper Score, Reock Ratio

The Polsby-Popper Score (PPS) is a metric used to quantify compactness. Recall that popular opinion is that compactness is something to be desired; the more compact a district is, the better. To calculate the PPS, we use the following definition:

$$PPS = \frac{4\pi(\text{Area of District})}{(\text{Perimeter of District})^2}$$

1. To get a sense of how the PPS measures compactness, we shall investigate the definition. What is the PPS for a circle of radius r ?

Takeaway: the PPS measures how different a district is from a circle with the same perimeter.

$$0 \text{ (more gerrymandered)} \rightarrow 1 \text{ (less gerrymandered)}$$

What is the PPS for a square district?

What is the PPS for a rectangular district that is four times as long as it is wide? Six times as long as it is wide?

4. Consider District Map W, and suppose each square measures 1 mile \times 1 mile.

a. Which district (1 through 7) appears to be the most compact?

b. Calculate the Reock Ratio for each district.

c. Which district is the most compact according to the Reock Ratio? In justifying your answer, state a hypothesis regarding the range of values one should expect to see from the Reock Ratio and how to interpret that ratio (do larger values correspond to more or less compact districts?).
