Teacher Exploration Key

PART I: THE "EFFECTIVENESS" OF DRUG TESTS

Assume a certain drug test is 98 percent accurate. This means 98 percent of people who used the given drug will test positive and 98 percent of the people who did not use the drug will test negative. Also assume that only 5 percent of people on the job (1 in 20) engage in drug use.

1. If a person tests positive, how likely is it that they actually used drugs? (Hint: To answer this question, you might consider a large population, such as 100,000 people. Figure out how many people in that population use drugs and how many users and nonusers test positive.)

Analyzing With Numbers

Students might analyze the situation by determining the number of people in each of the four categories for a fairly large sample, such as 100,000 people.

- For 100,000 people, there would be 5,000 drug users and 95,000 nonusers.
- Of the 5,000 users, 98 percent would test positive. This gives 4,900 true positives.
- Of the 95,000 nonusers, 2 percent would test positive. This gives 1,900 false positives.
- So, if someone tests positive, the probability that this person actually used drugs is 4,900 / 6,800, or about 72 percent.

This is considerably less than the phrase "98 percent accurate" might lead one to expect. Put another way, about 28 percent of people who test positive have not actually used drugs.

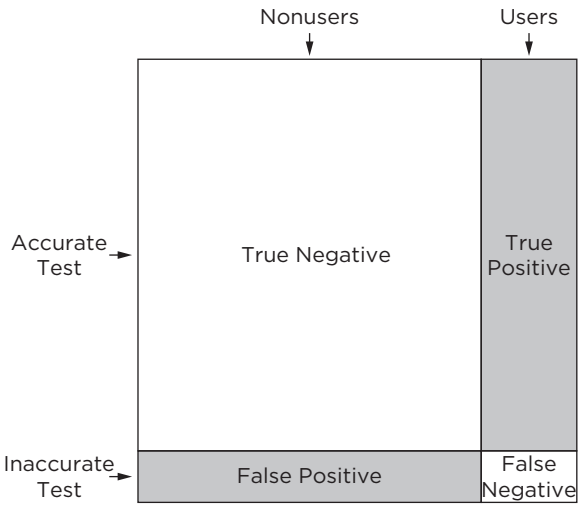
Analyzing With an Area Model

A diagram such as this might be helpful for understanding the various cases. As students work on developing their area model, you might ask:

- *What does each region on your diagram represent?*

Although this diagram is not to scale, it does illustrate several things.

- The nonusers heavily outnumber the users. Nonusers actually represent 95 percent of the total.
- For both the users and nonusers, those testing "true" outnumber those testing "false." Because the test is supposed to be 98 percent reliable, the "trues" are 98 percent of each group.
- There is a significant chunk of the population testing positive that did not actually use drugs (false positives).



Determining the exact area of each region will allow students to consider the fraction that answers, "If a person tests positive, how likely is it that they actually use drugs?" Ask students,

> *Of the 100,000 people, how many would be in each region of the area model?* (Or, if that has already been presented, *What connections are you all making between the calculations from before and this area model?*) *How might you use this diagram to answer the focus question for this task?*

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2. Do you think such a test should be used? Explain.

- Students may have diverse opinions. Some may emphasize the public safety importance of having certain workers be drug-free, while others may focus on the privacy issue. Ask, *What are some of the things you considered in deciding whether drug testing was fair?*
- Use the probabilities from Question 1 to emphasize the likelihood of a "false positive" result, and make sure students realize how severely a false positive could affect someone. For example, such a finding might result in someone being unfairly fired from a job. Ask, *How might we use our mathematical analysis to strengthen our arguments about the (un)fairness of drug testing?*
  + Students may suggest testing the positives a second time. If so, this could be a good chance for a mathematical extension to the problem. You could have groups calculate the probability that someone who tests positive a second time is indeed a true positive. (About 1 percent of those who test positive twice would actually be nonusers.)