Elicitation Strategy: Error Analysis

The **Error Analysis Strategy** is a strategy for gathering evidence of students' thinking, in which the teacher presents the class with a fictitious student response to a problem and asks them to figure out what the fictitious student did wrong or was reasoning about the problem. The fictitious response is designed to highlight a common student misconception that the teacher wishes to address with his or her students. It can be used both to probe the nature of students' thinking about the problem and to surface any similar misconceptions held by the students.

Particular Advantages

- Models the analysis of mistakes as a learning opportunity
- Often highlights a common misconception, creating an opportunity to discuss the misconception before it arises
- Provides a safe opportunity to assess student work since the work is anonymous and fictitious

How Does the Strategy Work?

1. Prior to using the strategy, the teacher identifies one or two errors for the class to analyze and finds or generates student work that illustrates the errors or misconceptions in the work.
2. Students first complete the problem on their own to gain familiarity with it. The teacher can choose whether to project the problem for the class or provide it individually on a handout.
3. The teacher then shows the class a fictitious student response and asks students to think about (or talk with a partner or small group about) the approach and reasoning shown in the response.
4. The class then discusses various ideas about the response. The goal of the conversation is to help students understand the nature of the errors or misconception and to be aware of it in their own work.

Example

For example, a teacher wishes to find out more about his or her students' understanding of comparison of fractions. The teacher knows that one common misconception is the overgeneralization of the idea that "the smaller the denominator, the larger the fraction." While this is true for unit fractions (i.e. $\frac{1}{3}$ is larger than $\frac{1}{4}$), students can sometimes extend this rule to all fractions, believing that $\frac{2}{5}$ is larger than $\frac{7}{9}$ because fifths are larger than ninths. So the teacher presents a fictitious student response that says:

> $\frac{2}{5} > \frac{7}{9}$. *If the denominator is bigger, the pieces are smaller. So the first fraction has bigger pieces.* $\frac{2}{5}$ *is bigger than* $\frac{7}{9}$.

He or she then says to the class:
This student is not correct, although some of his or her ideas are correct. What parts of the thinking are correct, and which parts are incorrect?

How Does the Strategy Support Formative Assessment?

Eliciting and interpreting evidence

- Teachers get information about the extent to which students may share the misconception provided in the response from the discussion that ensues, as students share their ideas about the fictitious response. Some students may pose additional questions, others may make assertions that correctly identify what was erroneous, and still others may make assertions that are themselves erroneous.

Student ownership and involvement

- Error Analysis helps students develop self-regulation skills as they practice analyzing the thinking and reasoning of others. As they gain experience with this kind of analysis, they can begin to apply it to their own mathematical reasoning.
- Error Analysis can boost students' confidence in their own reasoning by providing an acknowledgement that some misconceptions are based on thinking that is correct in certain mathematical settings but may have been applied to an incorrect mathematical setting—and thus gives credit for the validity of the initial thinking.

Environment

- Error Analysis also provides a learning environment where it is safe to be wrong and promotes the message that the teacher is interested in uncovering and understanding people's thinking rather than simply judging its accuracy.

How Might You Modify the Strategy, and Why?

For early attempts, consider showing and discussing one student's analysis of the work of another student. Once the class feels comfortable discussing errors together, you might ask students to share their own responses for discussion.

Students with organizational difficulties may benefit from a step-by-step organizer to help them go through an analysis process.

Elicitation Strategy: Flip-the-Question

Flip-the-Question is a strategy for posing mathematics questions to students to uncover their thinking. It can be used both to raise the level of cognitive demand of a math problem and to provide information to a teacher about the depth of a student's understanding.

Particular Advantages

- Provides information about students' understanding of the conceptual underpinnings of certain mathematics procedures
- Easy to implement, particularly for questions related to mathematical procedures
- Calls for higher order thinking skills

How Does the Strategy Work?

Start with questions of the form "If I give you <this information>, you calculate <this result>," and revise them into questions of the form "If you have <this result>, what information would you have started with?" Provide some time for students to think on their own before having them discuss their thoughts with a partner or as part of a full-class discussion.

Example

This strategy is best suited to questions that are focused on applying a procedure to calculate an answer. The revised question embeds the resulting correct answer in the question and asks students to think about what mathematical conditions could have produced that result.

For example:

Instead of asking: . . .	**. . . the teacher asks:**
What is the mean (average) of the numbers 7, 9, 11, 8.5, 7.5, and 5?	*What is a set of six numbers whose mean (average) is 8?*
What is the area of a triangle with a base of 10 and a height of 6?	*What is a possible base and height of a triangle with an area of 30 in^2?*
What are the common factors of 32 and 20?	*A pair of numbers has common factors of 2 and 4 only. What could the numbers be?*
$\frac{1}{3} \times \frac{3}{4} = ?$	*What pair of fractions could you multiply to get* $\frac{1}{4}$*?*

How Does the Strategy Support Formative Assessment?

Eliciting and interpreting evidence

- Students frequently need to draw on their conceptual understanding of the topic in order to answer the flipped question. Teachers can use this strategy to gather information about students' understanding of the conceptual underpinnings of a procedure.

- Because flipped questions often have more than one correct answer, they lead naturally into follow-up questions that provide a much broader picture of what a student knows. For example, a student responds that a possible base and height of a triangle with area 30 in^2 is 6 and 10. The teacher can then follow up with questions like "What is another base and height that also work?" or "Is that the only possible base and height?" These questions provide the teacher with a much fuller picture of the student's understanding of the area formula than seeing a student find the area of a triangle with base 6 and height 10.

Environment

- Flipping the question can often result in a mathematics question that is both attainable by most students (many of the questions could be explored with guess-and-check, even though that is not efficient), yet also can draw on higher order thinking skills as students analyze and test their ideas. These kinds of questions invite students to make the connections between mathematical ideas and to draw conclusions that are the punchlines of a lesson.

How Might You Modify the Strategy, and Why?

Broaden the flipped question to ask about the range of possible solutions. For example:

Original question:	**Flipped question:**	**Broadened question:**
What is the area of a triangle with a base of 10 and a height of 6?	*What is a possible base and height of a triangle with an area of 30 in^2?*	*How many possible whole-number bases and heights of a triangle with an area of 30 in^2 can you find?*
$\frac{1}{3} \times \frac{3}{4} = ?$	*What pair of fractions could you multiply to get $\frac{1}{4}$?*	*What are three different pairs of fractions that each have a product of $\frac{1}{4}$?*

What Are Some Considerations for Using the Strategy?

When using this strategy, be clear what mathematics you want to bring out from the flipped question and what you want students to learn from the problem posed. Not every problem must be modified, and it may be sufficient to flip the question for only one or two problems as the basis for an interesting mathematical discussion.