

Professional Development Guide

MATHEMATICAL ARGUMENTATION IN MIDDLE SCHOOL—THE WHAT, WHY, AND HOW

Introduction

The book is written for teachers to both read on their own and use together in a professional learning community (PLC). Working Together sections at the end of each chapter provide specific advice for teachers working in grade-level teams or PLC meetings or workshops. As a professional development (PD) leader, you can build on these sections to create a series of connected learning activities for teachers. This guide provides advice for doing so. It outlines eight sessions with an agenda, teacher activities, and facilitator notes.

First, we establish some overarching ideas.

PD NORMS

We recommend starting the teacher workshops with one or two improv games (found in Chapters 1–5 and 7 of the book) that will set up the following norms for how to behave in the workshop and support teachers in doing mathematical argumentation:

- Make bold conjectures.
- It's OK to be wrong.
- Find out the truth together.
- Build off other people's ideas.

DOING AND TEACHING ARGUMENTATION

We suggest mathematics tasks for teachers to do together that both introduce parts of argumentation and cover important standards-based content. It's crucial for teachers to have plenty of experience creating arguments for themselves, and that's why we include this component in every session. In most sessions, we include lesson planning, too.

TEACHING GAMES

The book is based on the PD that we have created over a 10-year period through three Bridging Professional Development projects. “Bridging” refers to making connections between the professional

learning activity and the classroom. We take seriously the idea of teaching as disciplined improvisation. This means that PD must provide teachers with knowledge—the discipline of the work—as well as an opportunity to try out new teaching moves, first in the safety of bounded games in PD, and then in the classroom. These teaching games are based on the games that improvisational theater performers play to learn their craft, but are about teaching. We adapted the games so that teachers can focus on using specific moves for specific purposes.

VISUALIZATIONS

Expert teachers visualize their lessons. We help beginning teachers do this kind of visualization through a structured process. The details of this process are in the book in Chapter 8 (see pages 142–143). In each session, we have suggested tasks from which teachers could choose, from each chapter, to plan and visualize. This is particularly useful if they are working through the activities from the book during the workshop. However, visualization should be something they will actually try out in the classroom. If teachers want to try visualization using their own lessons or curriculum, that could work too, as long as they are doing so for argumentation specifically.

PERFORMANCE

When we do PD on argumentation, we don't stop with lesson planning and visualization. We ask teachers to try out a part of their lesson in real time. We call this performance, because that's what improv actors do after learning their craft. One person takes the role of "teacher" and assigns "student" roles to his or her colleagues: the student who always wants to go first and the student who rarely contributes, for example. The "teacher" sets the scene: What happened in the lesson up to that point. Then they play out that part of the lesson together. "Students" should try to give the kinds of responses they think actual students will. The "teacher" can call time out at any time and get help from the facilitator or fellow teachers. He or she can also choose to "rewind" and try part of the lesson over.

In performance, teachers get to try out, in a low-consequence setting, moves they have planned for and to improvisationally use moves they hadn't expected to use. It's important to make performance a safe environment for all. Take on the improv norm "Make everyone look good." "Students" should not misbehave so much that no content gets covered. Ideas are offered in response to the questions the "teacher" has about his or her own performance, either by offering advice or critiquing what happened, in a safe, productive manner.

Here are our instructions for "teachers" for performance.

Before the performance:

- Select a 10-minute section of a lesson you have visualized. Choose a part you are unsure of. Decide which part of a learning goal you are focusing on.
- Prepare the mathematics task on a public display.
- Select roles for "students," such as the student who always wants to be right, the shy student, or the student with low computation skills.

To start the performance:

- Orient the "students" and the PD facilitator. What is the lesson? What has already happened? What segment are you going to focus on? Are "students" working in groups, or is it a whole-class discussion?

Perform!

- You and the “students” perform the lesson. You get to think on your feet in response to what “students” say. “Students” should stay in role and use what they know about what actual students may say in response to questions and prompts. Don’t forget to “make everyone look good.” We should assume good classroom management has led to cooperative, engaged “students” so that the focus is on the mathematics.

Reflect after the performance:

- What did you set out to do?
- Did it go as planned? Did you meet your goal?
- What mathematics did “students” get to do?
- What advice do you want from the group?
- Ask “students” how it felt to play their roles. Did they get insights into these types of students and their needs?

You can use performance as you see fit, once teachers become comfortable with visualizing their lessons. We also provide Session 8, which is devoted to performances.

ASSIGNMENT FOR BETWEEN SESSIONS

For each session, teachers should read the associated chapter before the session. We also provide an assignment in which teachers commit to trying out a new technique introduced in the session.

NUTS AND BOLTS

Your preparations should account for the following:

Timing

Each session is designed for a 6-hour day, but you may choose elements for half-day sessions. We have allotted 2 hours in each session for doing and teaching argumentation. It’s important to give plenty of time in your sessions for teachers to make arguments themselves. Knowing how to engage in argumentation is an important part of the knowledge base for disciplined improvisation. You may select additional tasks from the relevant chapters if more activities can be included in the allotted time. Making written plans is often included in this portion of the session and should also be given a generous amount of time, because visualization requires starting with written plans.

Materials Matter

Teachers should bring their own laptops or tablets. Still, provide each teacher with a spiral-bound graph paper notebook for their use in doing mathematics and taking notes during the sessions. We explain why graph paper is important for doing mathematics in the book. To help teachers internalize the norm that mistakes are OK, a nice touch is to provide erasable pens for their use. Make sure there are plenty of sticky notes, colored pencils and markers, and rulers.

Public Displays

Just as we ask teachers to plan for public displays of argumentation, you need to do so also. Depending on the size of your group, make sure to have large chart paper (preferably gridded), markers, a projector attached to a computer, and a document camera.

Online Space

Once, a thick binder full of paper materials including black-line masters was a staple of PD workshops. Now it is much easier to keep everything online where teachers can access it and notate it.

- Create an online space, such as a Google website linked to the Google drive with a folder for each teacher. Check with your school or district—they may have a preferred set of online productivity tools or courseware for you to use.
- Use the website or courseware to set up the agendas for the sessions as they appear in the book.
- Create online documents for relevant activities in the session. Link to them in the agenda. For example, provide a blank visualization lesson plan form (found at resources.corwin.com/mathargumentation) that teachers can make copies of to put in their own folders.

Prepare Yourself

Read the book, of course. Note in particular the planning methods in Chapter 8. You are going to use those in every session.

SESSION 1

Argumentation: What and Why

ONE-DAY AGENDA

Teachers read Chapter 1	In advance
Introductions and improv games: Zip, Zap, Zop and the Repeating Game	0.5 hour
Book chapter discussion	1 hour
Doing and teaching argumentation	2 hours
Teaching game: Open- and Closed-Ended Questions	1 hour
Visualization	1 hour
Wrap-up and assignment: Back in your classroom and Chapter 2	0.5 hour

INTRODUCTIONS AND IMPROV GAMES

Show the agenda and prepare teachers for the day's work. You may do introductions and other housekeeping.

TEACHER TASK	FACILITATOR NOTES
Play Zip, Zap, Zop from Chapter 1.	Discuss how these games might be useful in forming community—for them in the workshop and their students in class—and how they support argumentation—providing a similar framework and norms for participation.
Play the Repeating Game: <ul style="list-style-type: none">• The first person, the speaker, says a sentence about something in his or her life.• The second person, the repeater, may only repeat words that the first person says, with a questioning inflection.• Then the speaker continues talking, focusing on the repeater's question.• Keep going for a few minutes until conversation ends.• The facilitator yells, "Switch," and players switch roles so the speaker becomes the repeater.	

BOOK CHAPTER DISCUSSION

Teachers should read Chapter 1 in advance of this session. The discussion can be open-ended and based on their questions, or you can ask questions such as the following:

- What is mathematical argumentation? How is it different from argumentation in other subjects?
- What did you think of the advice in the Getting Started With Argumentation section? Can you see yourself taking the advice?
- How are the norms given in the poster Norms for Argumentation the same as or different from your current classroom norms?

DOING AND TEACHING ARGUMENTATION

Teachers should engage in making mathematical arguments. Chapter 1 provides a number of conjectures that teachers can work on justifying in this first session.

TEACHER TASK	FACILITATOR NOTES
<p>Convince yourselves: Why does the process for dividing fractions—the so-called invert and multiply—work?</p> <ul style="list-style-type: none"> Start with one example—say, $\frac{3}{4}$ divided by $\frac{1}{3}$ is $\frac{9}{4}$—and use a story or diagram to justify it. Then, using the example, discuss with your colleagues why the process always works. 	<p>If teachers need help, ask them the following questions:</p> <ul style="list-style-type: none"> What situation would require dividing $\frac{3}{4}$ by $\frac{1}{3}$? Should dividing $\frac{3}{4}$ by $\frac{1}{3}$ give the same or a different result as dividing $\frac{3}{4}$ by 3? Should the quotient be less than or equal to $\frac{3}{4}$? How do you know? Will this process work with all fractions? How do you know?
<p>Choose one of the conjectures in Chapter 1 and make your own justification for it.</p>	
<p>During the week, commit to asking at least twice in one lesson, “How do we know it is true?” Briefly write down how you plan to respond to the justifications that students give and guide them toward making stronger justifications.</p>	<p>Help teachers plan follow-up questions. They will learn more about this in the session on Chapter 4, so don’t expect sophistication at this point.</p>

TEACHING GAMES

The purposes of this pair of teaching games is to explore the affordances and drawbacks of open- and closed-ended questions, to see how each type of question creates a different effect on the students and the conversation.

TEACHER TASK	FACILITATOR NOTES
<p>Play the Closed-Ended teaching game.</p> <ul style="list-style-type: none"> Set up: Three to four teachers on each team. One teacher plays a “teacher”; the others play “students.” The rules: The “teacher” facilitates a conversation in which “students” justify the given conjecture. The “teacher” can use only <i>closed-ended questions</i> for which the answer is yes or no, or a number. Goal: Justify a conjecture. Reflect: How did that feel? When did it seem that the closed-ended questions moved the argument forward? When did they not? 	<p>You’ll need to use conjectures from the previous mathematics activities that were not used, or have prepared a list of unjustified conjectures for this game.</p> <ul style="list-style-type: none"> Play the game (about 5 minutes), until you can’t go on justifying. Ensure that the teachers are really playing it as a game. Have the “teacher” stand up and ask the questions of the “students.” Tell “students” to respond as themselves but only to what is required by the question.

TEACHER TASK	FACILITATOR NOTES
<p>Play the Open-Ended teaching game.</p> <ul style="list-style-type: none"> • Set up: Three to four teachers on each team. One teacher plays a “teacher”; the others play “students.” • The rules: The “teacher” facilitates a conversation in which “students” justify the given conjecture. The “teacher” can use only <i>open-ended questions</i> for which the answer consists of “student” ideas, and “students” use full sentences. • Goal: Justify a conjecture. • Reflect: How did that feel? When did it seem that the open-ended questions moved the argument forward? When did they not? 	<p>Debrief; Ask teachers for the affordances and drawbacks of closed-ended questions. Make sure they don’t jump to conclusions about the usefulness of these types of questions, but to consider the reasons why anyone might use open- or closed-ended questions.</p> <p>Ask if it’s possible to ask closed-ended questions that can lead to students making arguments. Ask if there are open-ended questions that don’t lead to students making arguments.</p>

VISUALIZATION

TEACHER TASK	FACILITATOR NOTES
<p>Discuss:</p> <ul style="list-style-type: none"> • What are teaching moves? • Which teaching moves did you learn about in Chapter 1? 	<p>Remind teachers about teaching moves from the book from Chapter 1.</p> <p>Say that the roles from the Repeater Game they played earlier will be useful during this activity.</p>
<p>Follow the instructions for visualization in Chapter 8.</p> <p>Observe one fishbowl visualization.</p>	<p>Have teachers use a lesson based on the dividing fractions task for visualization.</p> <p>Do one round of visualization fishbowl style, where one teacher volunteers to do the planning and you model listening, prompting for teaching moves, and taking notes in a lesson plan form on a public display. Do enough to give teachers an opportunity to see what the roles are and how to do it, but not too long or teachers will get restless.</p> <p>Debrief: What is the role of the visualizer? What is the role of the active listener/note taker?</p>

TEACHER TASK	FACILITATOR NOTES
Participate in a pair visualization.	Walk around the room, listening to how teachers are visualizing. Make sure the listening partner is engaged in active listening with the talking partner by taking notes and focusing on the moves and the expected student responses.
Report on the visualization.	Ask teachers to describe how the process felt to them. What did they learn from doing this process?

WRAP-UP

TEACHER TASK	FACILITATOR NOTES
Report one thing you learned today.	Gather a few quick reflections.

ASSIGNMENT

TEACHER TASK	FACILITATOR NOTES
<p>Write down two or three questions to ask your students over the next few days. At the next meeting, report to the group about how the questioning went in your classroom: What worked? What was frustrating? (10 min)</p> <p>During the week, commit to asking at least twice in one lesson, "How do we know it is true?" Briefly write down how you plan to respond to the justifications that students give and guide them toward making stronger justifications. (5 min)</p> <p>Read Chapter 2 from the book.</p>	Give a brief example of when the teachers might ask, "How do you know it is true?"
Quick evaluation: Write down one thing you learned today, what went well, and what you would change.	Review these reflections before the next session to see if you need to modify your agenda.

SESSION 2

Generating Cases

ONE-DAY AGENDA

Teachers read Chapter 2	In advance
Improv games and report from the classroom: Zip, Zap, Zop and First Letter, Last Letter	0.5 hour
Doing and teaching argumentation: Rectangle Coordinates	2 hours
Book chapter discussion	0.5 hour
Teaching game: Acting out a vignette	1 hour
Visualization	1 hour
Wrap-up and assignment: Try out generating cases with your students and read Chapter 3	0.5 hour

IMPROV GAMES

Show the agenda and prepare teachers for the day's work.

TEACHER TASK	FACILITATOR NOTES
Play Zip, Zap, Zop from Chapter 1.	Ask: <ul style="list-style-type: none">• What do these games have to do with argumentation?• What norms might you be eliciting with these games?
Play First Letter, Last Letter from Chapter 2.	

REPORT FROM THE CLASSROOM

Ask teachers to report on their assignment: Did they ask their students, “How do you know it’s true?” How did that go?

DOING AND TEACHING ARGUMENTATION

TEACHER TASK	FACILITATOR NOTES
Do Rectangle Coordinates together.	As teachers do the task, you will be modeling teaching moves for argumentation.
During this workshop, you should pay special attention to the teaching moves that support generating cases.	
	Do generation of cases and pattern finding in small groups.

and/or

TEACHER TASK	FACILITATOR NOTES
<p>Select a task for your grade level or use one of the following tasks from Chapter 2 to generate cases and find patterns.</p> <ul style="list-style-type: none">• For sixth grade: The signed numbers task• For seventh grade: The making triangles from given conditions task• For eighth grade: The transformational geometry task	<p>As the teachers work in their groups, you can ask the following questions:</p> <ul style="list-style-type: none">• How many cases did you examine, and what were the categories?• What were some of the special cases you considered or avoided?• What teaching moves—questions or prompts—would you use to generate additional cases?

BOOK CHAPTER DISCUSSION

In advance of the session, teachers should have read Chapter 2. The discussion can include questions about the reading and samples of teaching moves used in the classroom. You can ask,

- What does “generating cases” mean?
- Have you done anything similar to generating cases?
- What are some of the teaching moves you might use to support your students in generating cases?

TEACHING GAME

TEACHER TASK	FACILITATOR NOTES
<p>Enact the vignette in Chapter 2. Each teacher takes on a role and acts out the part. If you are the “teacher,” feel free to stand at the shared display. If you are a “student” who is putting forth an idea, maybe you also make the diagram.</p>	
<p>Write down a list of teaching moves that support students in generating cases.</p>	<p>Help take notes, maybe in a table, where you track the teaching move and the purpose. This is where an online shared document might be a helpful resource to make sure all voices are captured.</p>

VISUALIZATION

TEACHER TASK	FACILITATOR NOTES
Select one task (from Chapter 2) that you will use during the week to engage your students in generating cases. Respond to the cases that students give and guide them in making special or extreme cases.	If teachers need help, ask them the following questions: <ul style="list-style-type: none">• What are some of the moves you could use<ul style="list-style-type: none">◦ to encourage different groups of students to generate cases at their skill levels?◦ to push students to generate special or extreme cases?• What do you expect students to do or say after each move?
Follow the instructions for visualization in Chapter 8. Do one cycle of pair visualization.	
Report on the visualization exercise.	

WRAP-UP

TEACHER TASK	FACILITATOR NOTES
Describe one thing you learned today that would help students in generating cases.	Gather a few of the teachers' comments to share at the next session.

ASSIGNMENT

TEACHER TASK	FACILITATOR NOTES
Commit to teaching one lesson that will include having your students generate cases by working in pairs or small groups. Select two or three teaching moves for generating cases to try out with your students over the next few days. Read Chapter 3 from the book.	
Quick evaluation: Write down one thing you learned today, what went well, and what you would change.	Review these reflections before the next session to see if you need to modify your agenda.

SESSION 3

Conjecturing

ONE-DAY AGENDA

Teachers read Chapter 3	In advance
Improv games and report from the classroom: Zip, Zap, Zop and Gift Giving	0.5 hour
Doing and teaching argumentation	2 hours
Book chapter discussion	0.5 hour
Teaching game: Generating Conjectures	0.5 hour
Visualization	1 hour
Wrap-up and assignment: Students conjecturing in your class and read Chapter 4	0.5 hour

IMPROV GAMES

Show the agenda and prepare teachers for the day’s work. You may do introductions and other housekeeping.

TEACHER TASK	FACILITATOR NOTES
Play Zip, Zap, Zop from Chapter 1.	You may also try any other improv “ball games,” which you can find with a quick search online. Keep this first game quick—a reminder that these workshops are a safe space where mistakes are allowed and encouraged.
Play Gift Giving from Chapter 3.	Ask: <ul style="list-style-type: none">• What do these games have to do with argumentation?• Specifically, what does Gift Giving have to do with conjecturing?• What norms might you be eliciting with these games?

REPORT FROM THE CLASSROOM

Ask teachers to report on their assignment. Which questions from Chapter 2 did they try out with their students? How did it go?

Did they ask their students, “How do you know it’s true?” How did that go?

DOING AND TEACHING ARGUMENTATION

TEACHER TASK	FACILITATOR NOTES
<p>Make two to three conjectures using the Factoring activity in Chapter 3. Then discuss the following three questions:</p> <ul style="list-style-type: none"> • What are you absolutely sure of? • What are you pretty sure is true? • What could possibly be true? <p>Collectively, take one conjecture and justify it. Share your conjectures and your justifications with the whole group.</p>	<p>If teachers need additional help, ask the following questions:</p> <ul style="list-style-type: none"> • What patterns can you find? • Do you think they will hold for numbers greater than 20? How do you know? • How many factors does a prime number have? • Which numbers have the most factors? Is there a pattern? • What determines if a number has an odd or even number of prime factors?
<p>What types of conjectures will your students make? For instance, will they address a specific aspect of the content? Make conjectures that are false?</p> <p>As a grade-level group, design moves to elicit false conjectures for the activity on factoring.</p>	<p>After participants share their arguments for the Factoring task, have them work on Questions 4 and 5.</p>
<p>Through conjecturing, students can gain a deeper understanding of the mathematics; learn to use precise language; and communicate their understanding to others by speaking, writing, gesturing, and so on. Using the Factoring task, in your grade-level group, discuss the following questions:</p> <ul style="list-style-type: none"> • What is the mathematics vocabulary that can be developed through this task? • What do we already know for sure? • Given a pattern, do you think it holds for any case you could generate? If so, how can we describe that mathematically? If not, for which cases does it hold? • Can you use more mathematically precise language for stating the pattern? 	<p>If teachers need additional guidance, you can suggest the following:</p> <ul style="list-style-type: none"> • Is there another way to say your conjecture mathematically? • Do you think that the pattern holds for any case you could generate? If so, how can we describe that mathematically? If not, try to describe just the cases for which it does hold. • Can you use more mathematically precise language for stating the pattern?

BOOK CHAPTER DISCUSSION

Participants should read Chapter 3 ahead of time. The discussion can be open-ended and based on their questions, or you can ask the following questions:

- What did you think about the way students were introduced to the practice of conjecturing in the vignette?
- Would you use this approach with your students?

TEACHING GAME

TEACHER TASK	FACILITATOR NOTES
<p>Play the teaching game Generating Conjectures.</p> <ul style="list-style-type: none">• Set up: Three to four teachers on each team. One teacher plays a “teacher”; the others play “students.”• The rules: The “teacher” facilitates a conversation in which “students” generate conjectures for the Odd & Even Numbers task from Chapter 3.• Goal: Try out different ways of getting “students” to generate conjectures.• Reflect: What purposes did the moves you used serve?	<p>Do several 5-minute rounds so that all teachers in a group try teaching moves to support students’ efforts in making conjectures.</p> <ul style="list-style-type: none">• Encourage teachers to try moves they read about in the book.• Ensure that the teachers are really playing it as a game.• Have the “teacher” stand up and ask the questions of the “students.”• Tell “students” to respond as themselves but only to what is required by the question. <p>Debrief: Ask teachers to list the teaching moves that seemed to work the best to have students generate conjectures—that is, moves in which the students were doing the work of generating conjectures. Some of these could include “telling” students what conjectures are or reminding students of the norms for conjecturing.</p>
<p>Do another round, with another teacher playing the “teacher.” Do enough rounds so that everyone in a group gets to try out the “teacher” role.</p>	

VISUALIZATION

TEACHER TASK	FACILITATOR NOTES
<p>Select one task (from Chapter 3) that you will use during the week to engage your students in conjecturing.</p> <p>Respond to the cases that students give and guide them in making special or extreme cases.</p>	<p>If teachers need help, ask them the following questions:</p> <ul style="list-style-type: none"> • What are some of the moves you could use <ul style="list-style-type: none"> ◦ to encourage different groups of students to generate cases at their skill levels? ◦ to push students to generate special or extreme cases? • What do you expect students to do or say after each move?
<p>Follow the instructions for visualization in Chapter 8. Do one cycle of pair visualization.</p>	
<p>Report on the visualization exercise.</p>	

WRAP-UP

TEACHER TASK	FACILITATOR NOTES
<p>Report on one thing you learned in today's session.</p>	<p>Gather a few quick reflections.</p>

ASSIGNMENT

TEACHER TASK	FACILITATOR NOTES
<p>Select a lesson in which you plan for students to make conjectures during the next week. Plan to share your experiences at the next session.</p> <ul style="list-style-type: none"> • What worked well? • What changes will you need to make in your lesson? <p>Read Chapter 4 from the book.</p>	
<p>Quick evaluation: Write down one thing you learned today, what went well, and what you would change.</p>	<p>Review these reflections before the next session to see if you need to modify your agenda.</p>

SESSION 4

Justifying

ONE-DAY AGENDA

Teachers read Chapter 4	In advance
Improv games and report from the classroom: Zip, Zap, Zop and Story Spine	0.5 hour
Doing and teaching argumentation	2 hours
Book chapter discussion	0.5 hour
Teaching game: Facilitating Justification	0.5 hour
Visualization	1 hour
Wrap-up and assignment	0.5 hour

IMPROV GAMES

Show the agenda and prepare teachers for the day's work.

TEACHER TASK	FACILITATOR NOTES
Play Zip, Zap, Zop from Chapter 1.	You may also try any other “ball games,” which you can find with a quick search online. Keep this first game quick—a reminder that these workshops are a safe space where mistakes are allowed and encouraged.
Play Story Spine from Chapter 4.	Ask: <ul style="list-style-type: none">• What do these games have to do with argumentation?• Specifically, what does Story Spine have to do with justifying?• What norms might you be eliciting with these games?

REPORT FROM THE CLASSROOM

Ask teachers to report on their assignment: Which questions from Chapter 3 did they try out with their students? How did it go?

DOING AND TEACHING ARGUMENTATION

TEACHER TASK	FACILITATOR NOTES
<p>In your grade-level group, write a justification for a conjecture about the distributive property:</p> <p><i>$6x + 10$ is an expression without parentheses that is equivalent to $2(3x + 5)$.</i></p> <p>Discuss any connections you find across the different justifications in your group.</p>	<p>If teachers need help, ask them the following questions:</p> <ul style="list-style-type: none"> • How do the justifications show the same thing? • How do the visual representations for one justification help you understand the numbers or algebraic symbols in the other justification? • Which part of the argument do you agree with? Disagree with?
<p>Review the tasks in this chapter. As a group, select a conjecture based on one of the tasks and write a justification appropriate to your grade level.</p>	
<p>Plan a lesson for justification. Be sure to include possible conjectures for the lesson.</p>	<p>Help teachers plan for using teaching moves given in the book. They will further refine this through the teaching game and visualization.</p> <p>If teachers need help in planning the lesson, ask them the following questions:</p> <ul style="list-style-type: none"> • What are the learning goals? • How would the conjectures students make help them achieve those goals? • How would you order the conjectures for justification so that the justifications build on each other? • Which teaching moves will you need to support your students in justifying?

BOOK CHAPTER DISCUSSION

Teachers should read the chapter in advance of this session. The discussion can be open-ended based on their questions, or you can ask the following questions:

- What do you think are the goals of justification?
- What did you think about the moves for eliciting justifications?
- Can you see yourself using these moves in your classroom?

TEACHING GAME

TEACHER TASK	FACILITATOR NOTES
<p>Play the teaching game Facilitating Justification.</p> <ul style="list-style-type: none"> • Set up: Three to four teachers on each team. One teacher plays a “teacher”; the others play “students.” • The rules: The “teacher” facilitates a conversation in which “students” justify a conjecture for one of the tasks they did earlier. • Goal: Try out different ways of getting “students” to justify conjectures. • Reflect: What purposes did the moves you used serve? 	<p>Do several 5-minute rounds so that all teachers in a group try teaching moves to support students’ efforts in making justifications.</p> <p>Encourage teachers to try moves they read about in the book.</p> <p>Ensure that the teachers are really playing it as a game.</p> <p>Have the “teacher” stand up and ask the questions of the “students.”</p> <p>Tell “students” to respond as themselves but only to what is required by the question.</p>
<p>Do another round, with another teacher playing the “teacher.” Do enough rounds so that everyone in a group gets to try out the “teacher” role.</p>	<p>Make sure that moves for the following purposes get covered in the reflection:</p> <ul style="list-style-type: none"> • To get students started in justifying a conjecture, including one that may be false • To help students in developing their mathematical authority • To support students in using mathematical precision in communicating their ideas • To move students beyond justifying with a single example or case • To guide students in comparing justifications and critiquing the reasoning of others

VISUALIZATION

TEACHER TASK	FACILITATOR NOTES
<p>Use the lesson you planned and choose a particularly challenging part of the lesson. Follow the directions for visualization given in Chapter 8.</p>	<p>If teachers need help, ask them the following questions:</p> <ul style="list-style-type: none"> • What are some possible justifications your students would make? • What moves can you use to help elicit these justifications? • What moves can you use to get students to talk to each other?
<p>Report on the visualization exercise.</p>	<p>Ask teachers how active listening went.</p>

WRAP-UP

TEACHER TASK	FACILITATOR NOTES
Report on one thing you learned in today's session.	Gather a few quick reflections.

ASSIGNMENT

TEACHER TASK	FACILITATOR NOTES
<p>Select a lesson to teach that includes justifying based on one of the tasks in this chapter. After you teach the lesson, write out a few notes that you can use at the next meeting to help you reflect with your colleagues:</p> <ul style="list-style-type: none">• What moves did you use?• For what purposes?• How did your next steps (homework or exit tasks, for example) follow from the justifications students made? <p>Read Chapter 5 from the book.</p>	
Quick evaluation: Write down one thing you learned today, what went well, and what you would change.	Review these reflections before the next session to see if you need to modify your agenda.

SESSION 5

Representations in Justifications

ONE-DAY AGENDA

Teachers read Chapter 5	In advance
Improv games and report from the classroom: Zip, Zap, Zop and Magic Clay	0.5 hour
Doing and teaching argumentation	2 hours
Book chapter discussion	0.5 hour
Teaching game: Facilitating Justification	1 hour
Visualization	1 hour
Wrap-up and assignment	0.5 hour

IMPROV GAMES

Show the agenda and prepare teachers for the day's work.

TEACHER TASK	FACILITATOR NOTES
Play Zip, Zap, Zop from Chapter 1.	You may also try any other “ball games,” which you can find with a quick search online. Keep this first game quick—a reminder that these workshops are a safe space where mistakes are allowed and encouraged.
Play Magic Clay from Chapter 5.	Ask: <ul style="list-style-type: none">• What do these games have to do with argumentation?• Specifically, what does Magic Clay have to do with using visual representations in justifying?• What norms might you be eliciting with these games?

REPORT FROM THE CLASSROOM

Ask teachers to report on their assignment: What moves from Chapter 4 did they try out with their students? How did it go?

DOING AND TEACHING ARGUMENTATION

TEACHER TASK	FACILITATOR NOTES
<p>Convince yourselves: Why can a proportional relationship represented by an equation of the form $y = kx$, where k can be replaced by any number, be represented by a line? Must the line go through the origin? Why or why not? Start with a graph or table based on equations of the form $y = kx$. Then use the example to discuss why the process always works.</p>	<p>If the teachers need help, ask them the following questions:</p> <ul style="list-style-type: none"> • From the equation, what does it mean when you increase x by 1? • What happens in the table when the numbers in the x column are 1, 2, 3 . . . ? • Will the graph always be a straight line? How do you know?
<p>Review the task on dividing fractions. Use a visual representation to create a justification.</p>	<p>Remind teachers they have done this task before, and ask them to see if they can come up with a new justification based on a visual representation.</p>
<p>Plan a lesson for justification in which it is possible to use multiple representations.</p>	<p>Help teachers plan for using teaching moves given in the book. They will further refine this through the teaching game and visualization.</p> <p>If teachers need help in planning the lesson, ask them the following questions:</p> <ul style="list-style-type: none"> • What are the learning goals? • (if not using a premade conjecture) How would the conjectures students make help them achieve those goals? • How would you order the conjectures for justification so that the justifications build on each other? • Which teaching moves will you need to support your students in justifying using representations?

BOOK CHAPTER DISCUSSION

Teachers should read Chapter 5 in advance of this session. The discussion can be open-ended and based on their questions, or you can ask the following questions:

- What did you think about the use of representations to stimulate thinking about mathematical argumentation?
- Can you see yourself using these representations with your students?

TEACHING GAME

TEACHER TASK	FACILITATOR NOTES
<p>Play the teaching game Facilitating Justification using representations.</p> <ul style="list-style-type: none"> • Set up: Three to four teachers on each team. One teacher plays a “teacher”; the others play “students.” • The rules: The “teacher” facilitates a conversation in which pairs of “students” justify a conjecture using different representations. • Goal: Try out ways of connecting representations. • Reflect: What purposes did the moves you used serve? 	<p>Provide materials for making representations.</p> <p>Do several 5-minute rounds so that all teachers in a group try teaching moves that connect students’ representations.</p> <p>Encourage teachers to try moves they read about in the book.</p> <p>Ensure that the teachers are really playing it as a game.</p> <p>In the reflection, ask teachers which moves were used for what purposes:</p> <ul style="list-style-type: none"> • To elicit students’ representations • To help students in developing their mathematical authority • To move students beyond justifying with a single example or case • To guide students in comparing justifications and critiquing the reasoning of others
<p>Do another round, with another teacher playing the “teacher.” Do enough rounds so that everyone in a group gets to try out the “teacher” role.</p>	

VISUALIZATION

TEACHER TASK	FACILITATOR NOTES
<p>Use the lesson you planned and choose a particularly challenging part of the lesson.</p> <p>Follow the directions for visualization given in Chapter 8.</p>	<p>If teachers need help, ask them the following questions:</p> <ul style="list-style-type: none"> • What possible representations could students use to justify the conjectures? • How will you build on students’ representations? • How will you help students connect their representations?
<p>Report on the visualization exercise.</p>	<p>Ask teachers how active listening went.</p>

WRAP-UP

TEACHER TASK	FACILITATOR NOTES
<p>Report on one thing you learned in today’s session.</p>	<p>Gather a few quick reflections.</p>

ASSIGNMENT

TEACHER TASK	FACILITATOR NOTES
<p>Before the next session, commit to using a lesson that lends itself to multiple representations. Respond to the justification that students give and guide them toward making a stronger justification.</p> <p>Collect samples of student work that vary in their sophistication.</p> <p>Read Chapter 6 from the book.</p>	<p>Describe how you would provide a bridge to more formal mathematical expressions or concepts. What tools will you use?</p>
<p>Quick evaluation: Write down one thing you learned today, what went well, and what you would change.</p>	<p>Review these reflections before the next session to see if you need to modify your agenda.</p>

SESSION 6

Levels of Justification

ONE-DAY AGENDA

Teachers read Chapter 6	In advance
Improv games and report from the classroom: Gift Giving	1 hour
Book chapter discussion	0.5 hour
Doing and teaching argumentation	1.5 hours
Teaching game: Connecting Justifications	1 hour
Visualization	1 hour
Wrap-up and assignment	0.5 hour

IMPROV GAMES

TEACHER TASK	FACILITATOR NOTES
Play Zip, Zap, Zop from Chapter 1.	You may also try any other improv “ball games,” which you can find with a quick search online. Keep this first game quick—a reminder that these workshops are a safe space where mistakes are allowed and encouraged.
Play Gift Giving from Chapter 3.	Ask: <ul style="list-style-type: none">• What do these games have to do with argumentation?• What norms might you be eliciting with these games?• In particular, we use this game here because the student work we analyze is also like a gift.

REPORT FROM THE CLASSROOM

Have teachers share one of the samples of student work they collected as an assignment for the previous session and discuss the representations used.

BOOK CHAPTER DISCUSSION

Before this session, teachers should read Chapter 6 and assign one of the tasks to their students. The teachers should try using the four levels (0–3) of justification as a rubric to assess their students' work and bring one or two samples to share during the session. You can guide the discussion based on the teachers' questions, or start by asking the following questions:

- What did you think about the four levels for characterizing students' justifications presented in this chapter?
- How would you see yourself using these levels in the classroom?

DOING AND TEACHING ARGUMENTATION

TEACHER TASK	FACILITATOR NOTES
<p>Convince yourselves: The sum of any two consecutive numbers is odd. Start with a numeric example, say, $1 + 2 = 3$ or $2 + 3 = 5$, and then use representations, which can include variables, to justify it.</p> <p>How would you categorize the level of your justification?</p>	<p>If the teachers need help, ask them the following questions:</p> <ul style="list-style-type: none">• What situation is required to get a sum that is odd?• What patterns of sums are you observing?• How would you represent two consecutive numbers using a variable?• Should the algebraic expression for the sum of these two numbers also produce an odd number? How do you know?
<p>Present the samples of student work that you brought. Teachers should classify them based on the rubric in the book.</p>	<p>If teachers need help, ask them the following questions:</p> <ul style="list-style-type: none">• How did the student interpret justifying?• At which level was the student's response?• What did the student understand?• What did the student have difficulty understanding?• How did you help the student make a more sophisticated conjecture?

TEACHING GAME

TEACHER TASK	FACILITATOR NOTES
<p>Play a teaching game, Connecting Justifications, to connect justifications at different levels.</p> <ul style="list-style-type: none"> • Set up: Three teachers on each team. One teacher plays a “teacher”; the others play a pair of “students.” Each takes a justification from a teacher’s set of samples at different levels. • The rules: The “teacher” facilitates a conversation that builds the more sophisticated justification starting with the more basic one. • Goal: Help “students” make more sophisticated arguments. • Reflect: What purposes did the moves you used serve? 	<p>Provide materials for making representations.</p> <p>Do several 5-minute rounds so that all teachers in a group try teaching moves that connect students’ representations.</p> <p>Encourage teachers to try moves they read about in the book.</p> <p>Ensure that the teachers are really playing it as a game.</p> <p>In the reflection, ask teachers which moves were used for what purposes.</p>
<p>Do another round, with another teacher playing the “teacher.” Do enough rounds so that everyone in a group gets to try out the “teacher” role.</p>	

VISUALIZATION

TEACHER TASK	FACILITATOR NOTES
<p>Use the lesson from which student work samples came. Visualize supporting students who made a more basic justification moving to a more sophisticated one.</p>	<p>If teachers need help, ask them to look at the teaching moves for moving students’ justification from the basic level they started with.</p>
<p>Report on the visualization exercise.</p>	<p>Ask teachers how active listening went.</p>

WRAP-UP

TEACHER TASK	FACILITATOR NOTES
<p>Report on one thing you learned in today’s session.</p>	<p>Gather a few quick reflections.</p>

ASSIGNMENT

TEACHER TASK	FACILITATOR NOTES
<p>Think about what you will do differently as a result of analyzing samples of students' argumentation work. At the next session, prepare to share one of the changes you made in your classroom practice.</p> <p>Read Chapter 7 from the book.</p>	
<p>Quick evaluation: Write down one thing you learned today, what went well, and what you would change.</p>	<p>Review these reflections before the next session to see if you need to modify your agenda.</p>

SESSION 7

Concluding

ONE-DAY AGENDA

Teachers read Chapter 7	In advance
Improv games and report from the classroom: Word at a Time and String of Pearls	1 hour
Book chapter discussion	0.5 hour
Doing and teaching argumentation	2 hours
Teaching game: Concluding an Argument	1 hour
Visualization	1 hour
Wrap-up and assignment	0.5 hour

IMPROV GAMES

TEACHER TASK	FACILITATOR NOTES
Play Word at a Time and String of Pearls from Chapter 7.	Ask: <ul style="list-style-type: none">• What do these games have to do with argumentation?• Specifically, what do Word at a Time and String of Pearls have to do with <i>concluding</i>?• What norms might you be eliciting with these games?

REPORT FROM THE CLASSROOM

Have teachers share one of the changes they made in their teaching, as they were assigned in the previous session's assignment.

BOOK CHAPTER DISCUSSION

Teachers should read Chapter 7 in advance of this session. The discussion can be open-ended and based on their questions, or you can ask the following questions:

- What do you think about the different approaches for concluding an argument as presented in this chapter?
- Can you see yourself using these approaches in your classroom?

DOING AND TEACHING ARGUMENTATION

TEACHER TASK	FACILITATOR NOTES
Review the faulty arguments given in this chapter. Choose one and critique it to draw the correct conclusion.	
Plan a lesson that could elicit a typical faulty student argument for your grade level.	Ask teachers what teaching moves they can use to help students critique the argument.

TEACHING GAME

TEACHER TASK	FACILITATOR NOTES
<p>Play the teaching game Concluding an Argument. Choose a faulty justification for a false conclusion.</p> <ul style="list-style-type: none"> • Set up: Three to four teachers on each team. One teacher plays a “teacher”; the others play “students.” • The rules: The “teacher” facilitates a conversation in which “students” critique the given argument and then draw a different conclusion about the truth of the conjecture. • Goal: Try out moves for facilitating an alternative conclusion. • Reflect: What purposes did the moves you used serve? 	<p>Do several 5-minute rounds so that all teachers in a group try teaching moves for critiquing and concluding.</p> <p>Encourage teachers to try moves they read about in the book.</p> <p>Ensure that the teachers are really playing it as a game.</p> <p>In the reflection, ask teachers which moves were used for what purposes:</p> <ul style="list-style-type: none"> • To elicit a critique • To elicit a new argument • To elicit an alternative conclusion
Do another round, with another teacher playing the “teacher.” Do enough rounds so that everyone in a group gets to try out the “teacher” role.	

VISUALIZATION

TEACHER TASK	FACILITATOR NOTES
Use the lesson you planned, and choose a particularly challenging part of the lesson. Follow the directions for visualization given in Chapter 8.	If teachers need help, ask them the following questions: <ul style="list-style-type: none">• How will you summarize the argument?• Will the conclusions relate closely to content coming up in your curriculum?• Do the justifications rely on any new reasoning you want to point out to students?• What new conjectures could students make based on those they have concluded the truth or falsity of?• What new conjectures could you help students make based on the truth or falsity of your conclusion?
Report on the visualization exercise.	Ask teachers how active listening went.

WRAP-UP

TEACHER TASK	FACILITATOR NOTES
Report on one thing you learned in today's session.	Gather a few quick reflections.

ASSIGNMENT

TEACHER TASK	FACILITATOR NOTES
During the week before the next session, plan two to three target conjectures and teach an argumentation lesson that includes having students write a concluding statement and the argument for it. Respond to the conclusions that students give and guide them in formulating new conjectures.	Ask teachers how they will help students record their arguments in writing.
Quick evaluation: Write down one thing you learned today, what went well, and what you would change.	Review these reflections before the next session to see if you need to modify your agenda.

SESSION 8

Putting It All Together

ONE-DAY AGENDA

Improv games: Zip, Zap, Zop and String of Pearls	0.25 hour
Go over the rules for performance	0.25 hour
Prepare for performance	1 hour
Carry out performances for each teacher or pair of teachers	3.5 hours
Discussion: How has the book and these sessions changed the way we teach? What do we still need to learn?	1 hour

IMPROV GAMES

TEACHER TASK	FACILITATOR NOTES
Play Zip, Zap, Zop from Chapter 1. Play String of Pearls from Chapter 7.	Ask: <ul style="list-style-type: none">• What do these games have to do with argumentation?

RULES FOR PERFORMANCE

Read the section of the introduction to this PD guide on Performance for complete instructions. One person takes the role of teacher and assigns student roles to his or her colleagues: the student who always wants to go first and the student who rarely contributes, for example. He or she sets the scene: what happened in the lesson up to that point. Then they play out that part of the lesson together. “Students” should try to give the kinds of responses they think actual students will. The teacher can call time out at any time and get help from the facilitator or fellow teachers. He or she can also choose to “rewind” and try part of the lesson over.

PREPARE FOR PERFORMANCE

TEACHER TASK	FACILITATOR NOTES
Select a 10-minute section of a lesson you have visualized. Choose a part you are unsure of. Decide on the part of a learning goal on which you are focusing. Prepare the mathematics task on a public display. Select roles for “students,” such as the student who always wants to be right, the shy student, or the student with low computation skills.	Encourage teachers to choose a part of the lesson with which they want help, from you and other teachers. Provide different means of public display.

CARRY OUT PERFORMANCES AND REFLECT

TEACHER TASK	FACILITATOR NOTES
<p>Orient the “students” and the PD facilitator— what is the lesson? What has already happened? On what segment are you going to focus? Are “students” working in groups, or is it a whole-class discussion?</p> <p>Perform! You and the “students” perform the lesson. You get to think on your feet in response to what “students” say. “Students” should stay in role and also use what they know about what actual students may say in response to questions and prompts. Don’t forget to “make everyone look good.” We should assume good classroom management has led to cooperative, engaged “students” so that the focus is on the mathematics.</p>	<p>Be prepared to call “cut” on the performance to give feedback to the “teacher” if things seem to be going off track. Remind the “teacher” that he or she can call “cut” and ask for advice.</p> <p>Alternative: Have teachers teach in pairs rather than individually. This can be helpful if they seem tentative, or if time will not permit each teacher giving an individual performance.</p> <p>Try to start out with a strong performance so that teachers will have some confidence in the process. Be sure to set the model of calling “cut” for this performance.</p>
<p>Reflect after the performance:</p> <ul style="list-style-type: none"> • What did you set out to do? Did it go as planned? Did you meet your goal? • What mathematics did “students” get to do? • What advice do you want from the group? • Ask “students” how it felt to play their roles. Did they get insights into these types of students and their needs? 	<p>Guide teachers in answering the questions. Make it a group conversation rather than an individual reflection by the teacher.</p>

Retrieved from the companion website for *Mathematical Argumentation in Middle School—The What, Why, and How* by Jennifer Knudsen, Harriette S. Stevens, Teresa Lara-Meloy, Hee-Joon Kim, and Nicole Shechtman. Thousand Oaks, CA: Corwin, www.corwin.com. Copyright © 2018 by Corwin. All rights reserved. Reproduction authorized only for the local school site or nonprofit organization that has purchased this book.