Implementation Guide for Developing Criteria for Judging When an Argument is a Proof Activity

Task:	The Sum of Two Odds task, Analyzing Student Work Activity, Developing Criteria Activity
Goal:	To develop mathematical arguments (which may or may not constitute proofs) using a variety of representations, including visual, symbolic, and tabular. To compare arguments and discuss the criteria under which a mathematical argument counts as a proof.
Evidence:	Students are likely to construct a wide array of mathematical arguments, some of which (a set of empirical examples or a rationale) may not meet the criteria for a proof, and others of which (generic example, demonstration) would meet the criteria. Through the construction of an argument in their small group and a discussion across arguments, the notion that empirical arguments are not proof will be reinforced, and specific criteria for a proof can be developed.

SUM OF TWO ODDS TASK

Task

What is the main activity that students will be working on in this lesson?

Prove that when you add any two odd numbers, your answer is always even. Solve the task any way you want using any representations or materials that will help you make sense of the situation.

What are the various ways that students might complete the activity?

1. **An argument based on some number of numeric empirical examples.**
2. **An argument based on empirical examples with an accompanying visual representation of an odd number (e.g., pairs of blocks and one more).**
3. **A generic argument based on two specific examples (e.g., 5 and 11) with an accompanying visual representation that includes language that generalizes.**
4. **A symbolic proof of the sum of the *same* two odd numbers, represented as $2n + 1$ and $2n + 1$.**
5. **A symbolic proof of the sum of *any* two odd numbers, represented as $2m + 1$ and $2n + 1$.**
6. **An exhaustion argument, based on the fact that adding two odd numbers with ones digits 1, 3, 5, 7, or 9 will result in a number with a ones digit of 2, 4, 6, 8, or 0.**

Instructional Support

What tools or resources will students have to use in their work that will give them entry to, and help them reason through, the activity?

Task Sheet

Square blocks and/or graph paper

Chart paper (optional)

What questions might you ask students that will support their exploration of the activity and ***bridge*** *between* ***what they did*** *and* ***what you want them to learn*** *(the two shaded boxes)?*

1. How are you defining odd number? What different forms can we use to represent odd numbers?
2. How are you defining even number? What different forms can we use to represent even numbers?
3. For students with empirical arguments, ask what about their choices ensures that the outcome will be the same with a different pair of odd numbers.
4. (for students who struggle to get started) Can you solve a simpler problem, such as proving that the sum of any two single-digit odd numbers is even?
5. (for students with an argument like 4) If you choose a number for n, what are the two odd numbers you are adding?
6. How could your argument convince a skeptic? How could it convince an expert?

When arguments are complete, compare and discuss a range of arguments that vary with respect to the criteria for proof (show in the Evidence section)

Learning Goals (Residue)

What understandings will students take away from this activity?

Students will distinguish between empirical examples, rationales, and arguments that constitute proofs.

Students will identify criteria that make a mathematical argument for the conjecture compelling.

Evidence

What will students say, do, produce, etc., that will provide evidence of their understandings?

Students will identify features of arguments that justify the conjecture that they find compelling. These features should move toward the criteria for proof that will be discussed in the third lesson:

- Argument shows that the conjecture is true for all cases
- Statements in the argument are true and accepted by the mathematical community because they have been justified
- Conclusion follows logically from the argument made
- Mathematics is correct

ANALYZING MATHEMATICAL ARGUMENTS

Task

What is the main activity that students will be working on in this lesson?

Compare the 10 mathematical justifications for the Sum of Two Odds task. Identify which ones would count as proofs or non-proofs, and identify why as a small group.

What are the various ways that students might complete the activity?

1. **Students may find empirical arguments compelling.**
2. **Students may favor representations of one form as proof (e.g., symbolic) and dismiss others.**
3. **Students are likely to debate whether an argument contains enough language to be general (such as the distinction between B and C).**

Instructional Support

What tools or resources will students have to use in their work that will give them entry to, and help them reason through, the activity?

Student work A–J for the Sum of Two Odds task

What questions might you ask students that will support their exploration of the activity and ***bridge*** *between* ***what they did*** *and* ***what you want them to learn*** *(the two shaded boxes)?*

1. What about this argument makes you consider it a proof (or non-proof)?
2. What is the difference between B and C? Do the differences between the two matter?
3. What assumptions are made in this argument?
4. How do you know this argument holds for all integers?
5. Does it matter what representation is used?
6. (if not considered a proof) What additional information would you need to consider this argument a proof?

When small groups have completed their work in evaluating the arguments, share responses. Discuss work that most agree on and work that contains disagreements.

Learning Goals (Residue)

What understandings will students take away from this activity?

Students will distinguish between proof and non-proof arguments.

Evidence

What will students say, do, produce, etc., that will provide evidence of their understandings?

Students will agree on A, B, and J as proofs.

Students will agree on C, E, F, and H as non-proofs.

Students may have split opinions on D, G, and I (which are considered proofs). This need not be resolved as it will be addressed in the next lesson.

DETERMINING CRITERIA FOR PROOF

Task

What is the main activity that students will be working on in this lesson?

Using the responses from the student work analysis, determine the criteria needed for an argument to be considered a proof.

What are the various ways that students might complete the activity?

1. **Students may indicate that some representations are proofs and some are not.**
2. **Students may decide that *no* specific numbers can be used.**
3. **Students may consider a wide range of representational forms as proofs.**
4. **Students may see generality as arising from a wide range of arguments.**
5. **Students may wonder about the audience and whether that matters to be considered a proof.**

Instructional Support

What tools or resources will students have to use in their work that will give them entry to, and help them reason through, the activity?

Student work A–J for the Sum of Two Odds task

What questions might you ask students that will support their exploration of the activity and ***bridge*** *between* ***what they did*** *and* ***what you want them to learn*** *(the two shaded boxes)?*

1. Why does the audience matter for a proof?
2. How do we determine whether a statement is true? Who decides?
3. Is a proof different for us as compared to a fifth grader? A college math major? A mathematician?
4. How "formal" does the language have to be to be considered a proof?
5. Does a proof have to have a specific structure?

When small groups have drafted a set of criteria, have them share and aggregate common themes across them that correspond to the criteria in the Evidence section.

Learning Goals (Residue)

What understandings will students take away from this activity?

Students will identify the criteria that make a valid mathematical argument a proof.

Students will understand that valid proofs can take different representational forms.

Evidence

What will students say, do, produce, etc., that will provide evidence of their understandings?

Students will identify the criteria for proof as follows:

- Argument shows that the conjecture is true for all cases
- Statements in the argument are true and accepted by the mathematical community because they have been justified
- Conclusion follows logically from the argument made
- Mathematics is correct