Criteria for Judging Whether an Argument Counts as a Proof

An argument that counts as a proof must meet all of the following criteria:

1. **The argument must show that the conjecture or claim is (or is not) true for *all* cases.** *(Student H is an example of making an argument about all cases of sums of two odd numbers. While Student C also provides an algebraic argument, it deals only with the special case of adding any two of the same odd numbers, so it does not meet this criterion.)* Specific examples *can* be used in the argument, but it is essential at some point to move from particular examples to a discussion of the more general case. *(While both Students A and E use a specific example, only Student A moves from the specifics of adding 5 + 11 to the more general case of considering any two odd numbers. Therefore, Student A's argument would be considered a proof but Student E's argument would not.)*

2. **The statements and definitions that are used in the argument must be ones that are true and accepted by the community because they have been previously justified.** *(Student F used the statement that even + even = even in her proof that odd + odd = even. Since she goes on to say that this statement had been previously established, it was appropriate to use. The first statement made by Student B is false, and therefore anything that follows from this should be suspect.)*

3. **The conclusion that is reached from the set of statements must follow logically from the argument made.** *(Student H's conclusion that* X + Y *is an even number follows logically from the set of statements made beginning with defining* X *and* Y *to represent odd numbers 2*n + *1 and 2*m + *1, respectively; Student F's conclusion that "if you add two of them together, you get an even number + 2, which is still even" follows from the argument made regarding an odd number being the sum of an even number and 1.)*

4. **The mathematics must be correct.** *(Among other problems with Student B's response, there are mathematical mistakes in this argument. For example, Student B writes that 5*a + *5*b = *10(*a + b*), which is incorrect.)*

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