

# What Does Concept-Based Curriculum Look Like in the Math Classroom?

## Main Text: Chapter 1. Why Is It Important for My Students to Learn Conceptually?

What are the facts, processes, and concepts in mathematics? Mathematics is an inherently conceptual language, so both the Structure of Knowledge and the Structure of Process apply to mathematics. What are concept-based curricula?

Concept-based is not a curricular program, but rather a design model that can overlay any curricula to enhance conceptual depth and understanding. Concept-based encourages the explicit delineation of what students must Know (factually), Understand (conceptually), and be able to Do (skill) within the particular school's curricula. The traditional objectives-based model of curriculum design failed to clearly differentiate these facets (KUDs). (Erickson & Lanning, 2014)

In this module you will find the following resources:

- Examples of the Structure of Knowledge and the Structure of Process using linear functions
- A summary of the Structure of Knowledge and the Structure of Process
- A blank template of the Structure of Knowledge for you to use when you think about your own example
- An example of a traditional activity versus an activity that promotes synergistic thinking
- The synergistic thinking task template
- An outline of the five tenets of concept-based math curriculum and instruction
- Additional sample conceptual lenses relevant for mathematics
- Discussion questions for Module 1
- An opportunity for reflection

FIGURE M1.1: THE STRUCTURE OF KNOWLEDGE FOR LINEAR FUNCTIONS

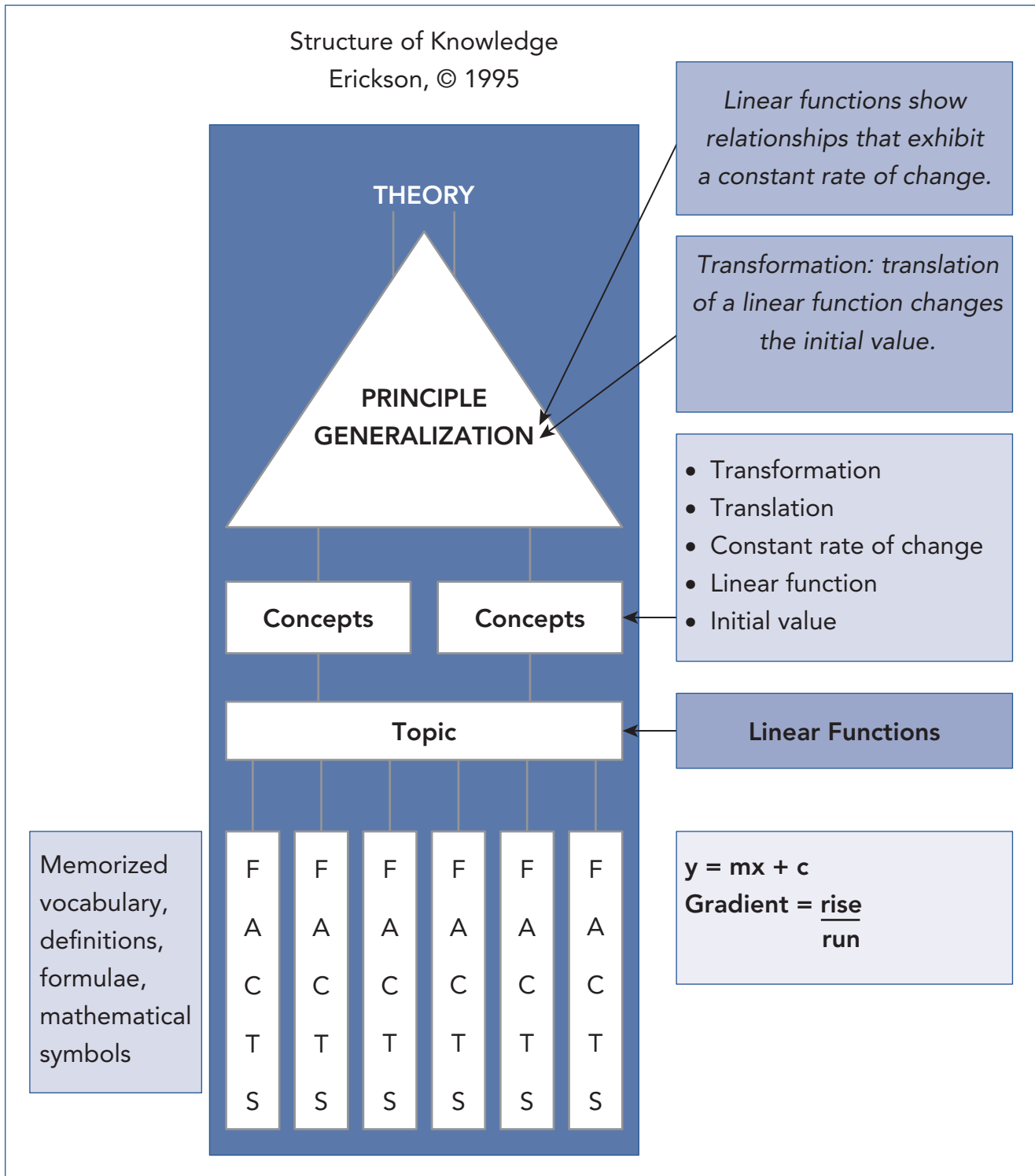
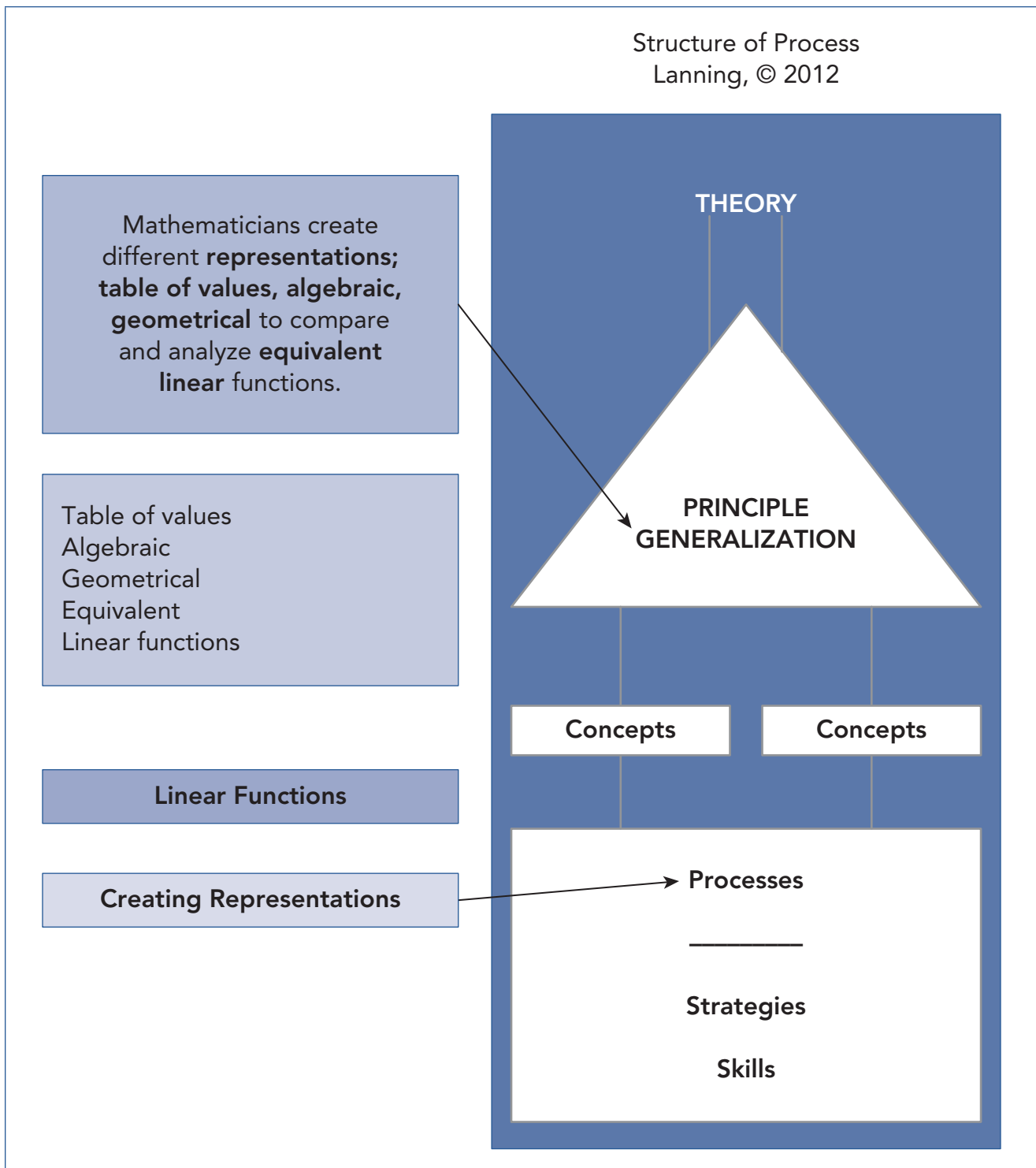


FIGURE M1.2: THE STRUCTURE OF PROCESS FOR LINEAR FUNCTIONS



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FIGURE M1.3: THE STRUCTURE OF KNOWLEDGE

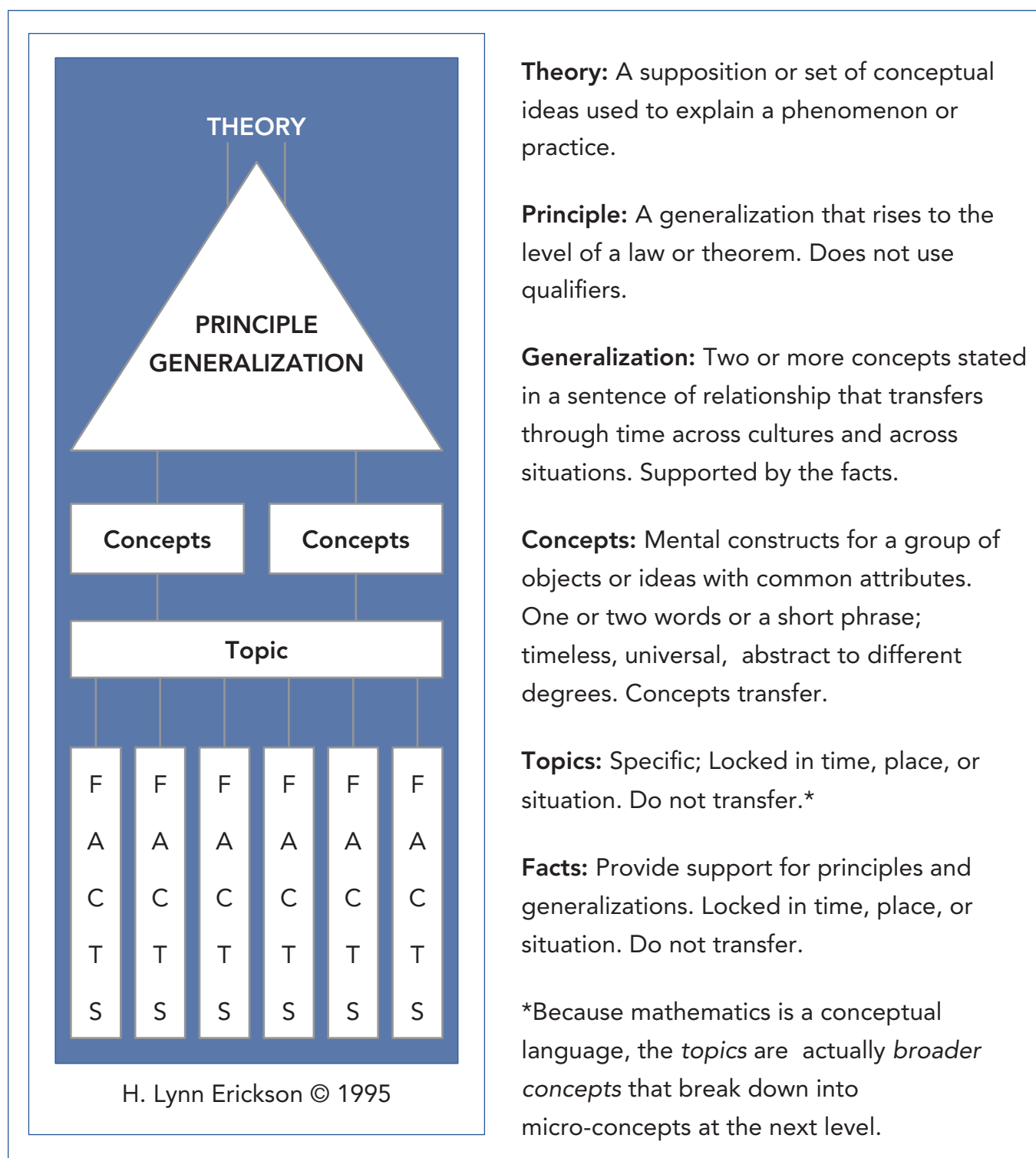


FIGURE M1.4: THE STRUCTURE OF PROCESS

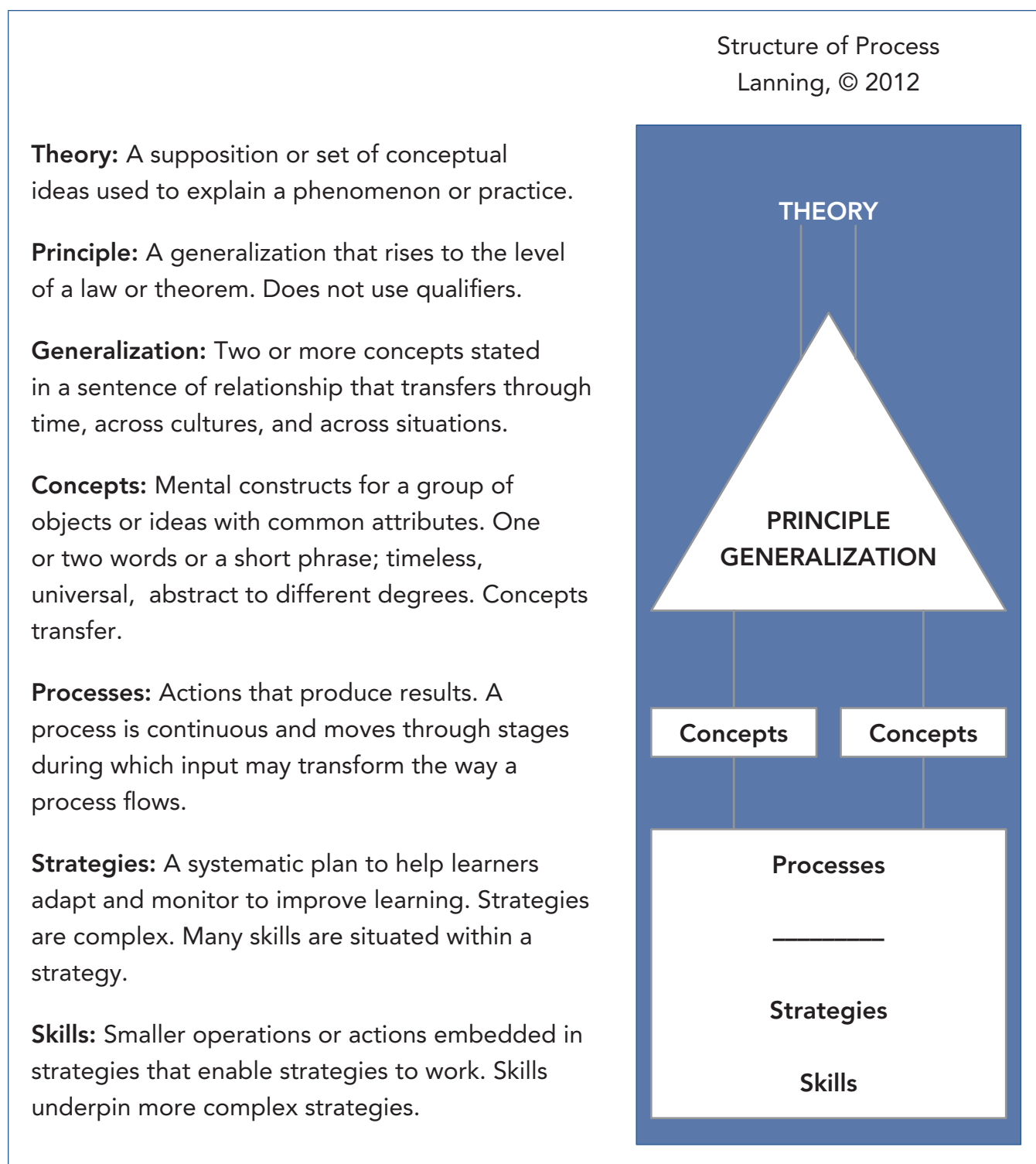
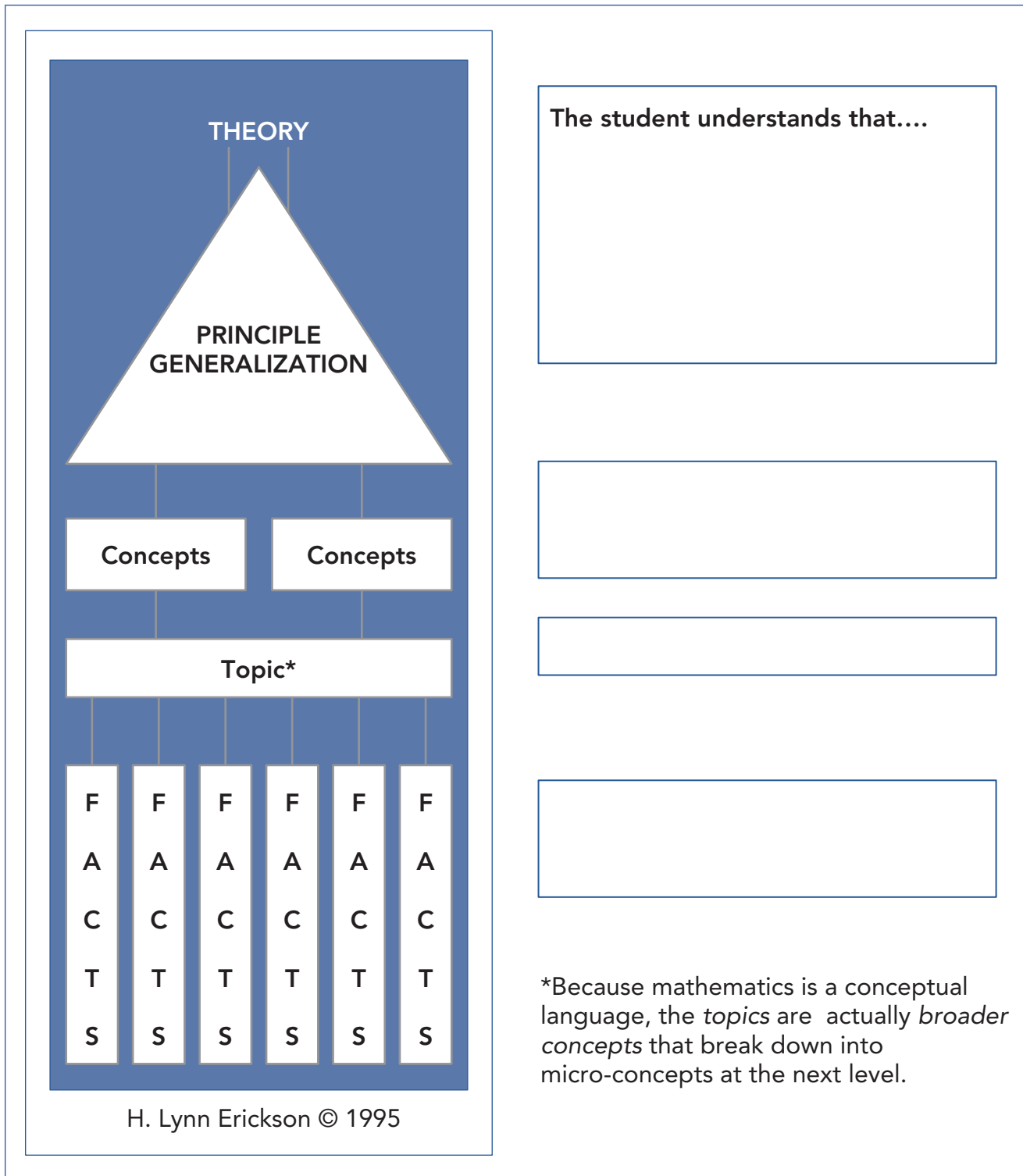


FIGURE M1.5: TEMPLATE FOR THE STRUCTURE OF KNOWLEDGE



\*Because mathematics is a conceptual language, the *topics* are actually *broader concepts* that break down into *micro-concepts* at the next level.

FIGURE M1.6: EXAMPLE OF SYNERGISTIC THINKING WHEN PLANNING TASKS

**Synergistic thinking means that . . .**

**Traditional Activity:**

Use a table of values to plot the following:  $y = 3x + 4$ ;  $y = 3x + 5$ ;  $y = x - 2$ ;  $y = x + 1$

**Adapted to facilitate synergistic thinking:**

**Coordinates Game (see Chapter 4, Figure 4.8 for full details)**

Ask student in position bottom left to be the coordinate (0,0). Now ask all students to work out their position and write their coordinates. Do not tell students what their coordinate is.

Ask students to stand up if they are  $x = 0$ ,  $x = 1$ , then  $x = 2$ , then  $x = 3$  and  $x = 4$ .

Next, go through  $y = 0, 1, 2, 3, 4$ .

Now show the card  $y = x$  and ask students to stand up if they follow this rule.

Next show the cards  $y = 2x, 3x$ , and  $4x$ .

Now show  $y = x + 1$  and ask students to stand up if they follow this rule. Then  $y = x + 2$ ;  $y = x + 3$ .

**Option:** Ask students to change positions, and ask them to work their coordinates and repeat the same linear equations.

FIGURE M1.7: TEMPLATE FOR SYNERGISTIC THINKING WHEN PLANNING TASKS

**Synergistic thinking means that . . .**

**Traditional Activity:**

**Adapted to facilitate synergistic thinking:**



FIGURE M1.8: THE TENETS OF CONCEPT-BASED CURRICULUM AND INSTRUCTION

Explain each of the tenets of concept-based curriculum and instruction and give examples to illustrate your explanations.

1. Synergistic thinking
2. The conceptual lens
3. Inductive teaching
4. Guiding questions
5. Three-dimensional curriculum and design model

FIGURE M1.9: THE CONCEPTUAL LENS IN MATHEMATICS

A conceptual lens is a broad, integrating concept that focuses a unit of work to allow students to process the factual information, for example, the study of patterns when looking at different types of sequences, such as square and triangular numbers.

Sample Conceptual Lenses for Mathematics

identity	form	errors	randomness
change	logic	parts and wholes	balance
systems	relationships	variation	representation
transformations	uncertainty	precision	order
patterns	space	structure/function	quantity
accumulation	development	abstraction	intuition

Adapted from Lynn Erickson (2007).

## Discussion Questions for Module 1

1. Create your own math examples for the Structure of Knowledge and the Structure of Process.
2. What are some examples of inductive teaching?
3. How do you use the conceptual lens to focus a unit of work? Choose one from the list in Figure M1.9 and provide an example of how you would use it.
4. Can you think of more conceptual lenses for math?
5. How do you promote synergistic thinking in your classroom?
6. How do you apply the philosophy and design model of concept-based curriculum to your own curriculum?

## An Opportunity for Reflection

**I used to think . . .**

**Now I think . . .**

**I understand that . . .**

# How Do Teachers Write Quality Generalizations? What Do We Want Our Students to Understand From Their Program of Study?

## Main Text: Chapter 3. What Are Generalizations in Mathematics?

Generalizations give us explanations of why and what we want our students to comprehend in terms of the relationship between two or more concepts. Also known in education circles as *enduring understandings*, *essential understandings*, or *big ideas of the unit*, they summarize what we would like our students to take away from their unit of study. It is important when crafting generalizations to not merely write definitions or objectives. Effective mathematics learning arises out of guiding students to particular principles or generalizations through inductive inquiry. Each unit of work contains six to eight generalizations.

This module contains the following resources:

- A table of verbs to avoid when crafting generalizations
- An exemplar of the scaffolding process when crafting generalizations
- A template for scaffolding generalizations
- A table of sample verbs for scaffolding level 2 and 3 generalizations
- A checklist for crafting generalizations
- Discussion questions for Module 2
- An opportunity for reflection

When crafting generalizations, remember these guidelines:

- Don't just write definitions of terms
- Use your suggested sample verbs list
- The goal is to help students deeply understand as a result of studying the unit
- No pronouns, past tense verbs, proper nouns, or “no no” verbs such as *is*, *are*, *affects*, *impacts*, *influences*

- Ask yourself the following questions:
  - Do the ideas grow in sophistication?
  - Do the ideas become clearer at Level 2 because they are more specific (use more specific concepts)?
  - Did I answer the question at each level? (Level 2 How or why? Level 3 So what is the significance?)
  - Are the verbs active and present tense?
  - Are the ideas based in fact? (Are they true?)
  - Are the ideas important?
  - Are the ideas developmentally appropriate?
- Start off with this stem: Students will understand that . . .

The generalization equation:

$$\text{Generalization} = \text{concept 1} + \text{strong verb} + \text{concept 2}$$

(Generally you could have more than two concepts, but the concepts need to be compatible when scaffolding.)

Lynn Erickson (2007) states that avoiding certain “no no verbs” will strengthen the expression of a generalization (see Chapter 3). Take a look at the following Level 1 generalizations:

*The gradients of two parallel lines are equal.*

*The area of a circle is found by multiplying the radius squared by  $\pi$ .*

*Speed is affected by distance and time.*

*Multiplying two negative numbers equals a positive.*

Because they are constructed using “no no verbs,” the above statements do not reflect an understanding of why or how these concepts are significant.

Below is a table of “no no verbs” adapted from the work of Lynn Erickson.

FIGURE M2.1: NO NO VERBS WHEN CRAFTING GENERALIZATIONS

is	are	have	influences
affects	impacts	implies	equals

Adapted from Lynn Erickson (2007)

During the process of forming your generalizations (enduring understandings) for any unit, start off with the sentence “After this program of study, students will understand that . . .” Often initial ideas result in a Level 1 generalization that includes “no no” verbs from Figure M2.1. To develop the quality of your conceptual statement to the next level, ask the questions *How?* or *Why?* Often this will provide a Level 2 generalization that can be used in the unit of work. Sometimes generalizations can scaffold to Level 3 by asking the question *So what?*

Generally a unit of work will have six to eight generalizations, which are a combination of Levels 2 and 3. The teaching target is the Level 2 generalization; use Level 3 for extension or clarification.

Here are a couple of examples of the scaffolding process:

FIGURE M2.2: SCAFFOLDING THE PYTHAGOREAN THEOREM

<b>Level 1</b>	<p><b>Students will understand that . . .</b></p> <p><i>Two side lengths of any right triangle affect the third side.</i></p>	The teaching target in Level 2 generalization
<b>Level 2</b>	<p><b>How or why?</b></p> <p><i>For right-angled triangles, the area of the square drawn from the hypotenuse represents the sum of the areas of the squares drawn from the other sides.</i></p>	
<b>Level 3</b>	<p><b>So what? What is the significance or effect?</b></p> <p><i>For right-angled triangles, knowing the area of the square drawn from the hypotenuse or an area drawn from one of the other sides allows calculation of an unknown side.</i></p>	

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FIGURE M2.3: SCAFFOLDING GENERALIZATION FOR SEQUENCES AND SERIES

The teaching target in Level 2 generalization

<b>Level 1</b>	<b>Students will understand that . . .</b> <i>Arithmetic and geometric sequences and series are structured by a set of formulae and definitions.</i>  This generalization contains a “passive voice” verb, which weakens the sentence. Flipping the sentence emphasizes the strong verb for a Level 2 generalization. This also contains a no verb.
<b>Level 2</b>	<b>How or why?</b> <i>A set of formulae and definitions structure arithmetic and geometric sequences.</i> <i>Whether arithmetic and geometric sequences and series share a common difference or a common ratio distinguish one from another.</i>
<b>Level 3</b>	<b>So what? What is the significance or effect?</b> <i>Arithmetic and geometric sequences and series describe patterns in numbers supply algebraic tools that help to solve real-life situations.</i>

The following is a scaffolding template to support the crafting of generalizations developed by Lynn Erickson (2007, p. 93).

**FIGURE M2.4: SCAFFOLDING GENERALIZATIONS TEMPLATE**

<b>Meso Concept: Micro Concepts: (see Chapter 3)</b>
Level 1 Generalization: After this program of study, students will understand that . . .
Level 2 Generalization: Why or how?
Level 3 Generalization: So what? What is the significance? What is the effect?
Level 1 Generalization: After this program of study, students will understand that . . .
Level 2 Generalization: Why or how?
Level 3 Generalization: So what? What is the significance? What is the effect?



Here are some Level 2 and 3 verbs to help support the process of writing generalizations.

**FIGURE M2.5: SAMPLE VERBS TO USE WHEN CRAFTING GENERALIZATIONS**

accelerate	compile	draw up	gather
accomplish	compose	drive	generate
achieve	compute	earn	give
acquire	conceive	edit	govern
activate	condense	elaborate	guide
adapt	conduct	eliminate	handle
address	conserve	empathize	help
adjust	consolidate	employ	hypothesize
administer	construct	enact	identify
advance	contribute	encourage	illustrate
allocate	convert	enforce	imagine
analyze	convey	engineer	implement
anticipate	cooperate	enhance	improve on
approve	coordinate	enlist	improvise
arrange	correlate	ensure	increase
ascertain	correspond	equip	inform
assemble	create	establish	initiate
assess	cultivate	estimate	innovate
assign	deal	evaluate	inspect
assimilate	decide	examine	inspire
assist	define	execute	install
assure	delegate	expand	instill
attain	deliver	expedite	institute
attend	demonstrate	experiment	instruct
balance	design	explain	insure (ensure)
bring	detect	express	integrate
bring about	determine	facilitate	interpret
build	develop	fashion	introduce
calculate	devise	finance	invent
challenge	direct	fix	investigate
chart	discover	follow	judge
check	display	forecast	justify
clarify	distribute	forge	reconcile
classify	document	form	record
collect	draft	formulate	recruit
command	dramatize	function as	rectify
communicate	draw	gain	keep

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kindle	pilot	render	study
launch	pioneer	reorganize	substitute
lead	place	repair	succeed
learn	plan	report	summarize
lift	play	represent	supersede
listen to	predict	research	supervise
locate	prepare	resolve	supply
maintain	prescribe	respond	survey
make	present	restore	symbolize
manage	prevent	retrieve	synergize
manipulate	problem solve	revamp	talk
market	process	review	teach
master	procure	revise	tell
mediate	produce	revitalize	tend
meet	project	revive	test
memorize	promote	rout	trace
mentor	propose	save	track
minimize	protect	schedule	translate
model	prove	screen	travel
monitor	provide	secure	treat
motivate	publicize	select	trim
move	purchase	sell	train
navigate	question	sense	transcribe
negotiate	raise	separate	transfer
nominate	read	serve	transform
observe	realize	service	uncover
obtain	reason	show	undertake
offer	receive	simplify	unify
optimize	recognize	sketch	unite
orchestrate	recommend	solidify	update
order	set up	solve	upgrade
organize	shape	sort	use
originate	share	spark	utilize
overcome	shift	speak	validate
oversee	redesign	spearhead	verify
paint	reduce	staff	widen
participate	re evaluate	stimulate	win
perceive	refer	streamline	withdraw
perfect	refine	strengthen	work
perform	regulate	stress	write
persuade	relate	stretch	
photograph	remember	structure	

FIGURE M2.6: CHECKLIST FOR CRAFTING GENERALIZATIONS

Are there no no verbs?	
Do some of the generalizations in the unit answer the questions <i>How?</i> or <i>Why?</i>	
Do some of the generalizations in the unit answer the <i>So what?</i> question?	
Have you included approximately six to eight generalizations in your unit?	
Have you thought about how you are going to draw the generalizations from your students?	
Are your generalizations the most important big ideas, the enduring understandings, and what you would like students to understand from their program of study?	
Are your generalizations statements that include two or more concepts in a related context?	
Would you be happy if your students expressed this generalization to you?	

## Discussion Questions for Module 2

1. Explain why the “no no” verbs result in weak generalizations.
2. What is the purpose of scaffolding generalizations?
3. What is the difference between a Level 1, 2, and 3 generalization?
4. How will you develop quality generalizations?
5. Provide five Level 2 or 3 verbs not on the list in Figure M2.5.
6. How will you draw generalizations from your students?
7. Which of these generalizations best describes the purpose of the quadratic formula? Explain the reason for your choice. Will your students choose the same one as you? Why or why not?
  - The quadratic formula is  $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ .
  - Solving a quadratic equation by using the quadratic formula graphically displays the  $x$  intercepts of a quadratic function.
  - The quadratic formula tells us what  $x$  is equal to and allows us to solve quadratic equations.
8. Can you think of other ways that you can guide your students toward the generalizations in a unit of work?

## An Opportunity for Reflection

**3 things I learned:**

**2 things I found interesting:**

**1 thing I need to develop:**

# Guiding Questions and Critical Content

## How Can I Draw Essential Understandings From Students?

### Main Text: Chapter 4. How Do I Plan Units of Work for a Concept-Based Curriculum?

Guiding and essential questions guide students' thinking from the concrete to the conceptual level. When planning your unit of work, one of the critical content components is the guiding or essential questions. Guiding questions bridge the factual to the conceptual, which supports the generalizations of the unit. The questions can be categorized into three types: factual, conceptual, and debatable (or provocative). For each unit of work there may be six to eight generalizations, and each generalization may have three to five factual and conceptual questions. For the entire unit there may be two to three debatable or provocative questions. Concept-based curriculum design is also focused on what students will know (factually), understand (conceptually), and be able to do (in terms of skills). These are known as the KUDs of the unit. What we want our students to understand is summarized by the generalizations of the unit.

In this module you will find the following resources:

- An example of the three types of guiding questions
- A template for designing the three types of guiding questions—factual, conceptual, and provocative/debatable
- An example of how to use KUDs
- A template to plan your KUDs for a unit of work
- Discussion questions for Module 3
- An opportunity for reflection

FIGURE M3.1: EXAMPLE OF GUIDING QUESTIONS FOR GEOMETRY: ORDERED PAIRS

<div style="border: 1px solid black; padding: 5px; display: inline-block;"> <b>Unit of Study:</b>            Geometry         </div>	
<h2 style="margin: 0;">Guiding Questions</h2>	
<p><b>Generalization:</b></p> <p><i>Ordered pairs define the relationship between values on a coordinate plane that describe real situations.</i></p>	
<p><b>Factual Questions</b> What?</p>	<p>How do you plot <math>(x, y)</math> on a coordinate plane?</p> <p>How do you draw an <math>x</math> and a <math>y</math> axis?</p> <p>What coordinates are given by the origin?</p>
<p><b>Conceptual Questions</b> Why or How?</p>	<p>How are coordinates useful in real life?</p> <p>Why is the order important when naming coordinates?</p> <p>How do coordinates such as <math>(1, 2)</math>, <math>(2, 4)</math>, and <math>(3, 6)</math> express an <math>x, y</math> relationship?</p> <p>Why can <math>x</math> and <math>y</math> be used to represent real-life situations?</p> <p>Why is the coordinate plane known as a <i>plane</i>?</p>
<p><b>Debatable/Provocative Question</b></p>	<p>Are points on the coordinate plane one- or two-dimensional?</p>

FIGURE M3.2: EXAMPLE OF GUIDING QUESTIONS FOR GEOMETRY: TWO-DIMENSIONAL SHAPES

<div style="border: 1px solid black; padding: 5px; display: inline-block;"> <b>Unit of Study:</b>                  Geometry             </div>	
<h2 style="margin: 0;">Guiding Questions</h2>	
<p><b>Generalization:</b>  <i>Properties of two-dimensional plane shapes allow mathematicians to classify and compare common attributes.</i></p>	
<p><b>Factual Questions</b>                  What?</p>	<p>What are the properties of a quadrilateral?                      What are the properties of a square?                      What are the properties of a rectangle?                      What are the properties of a parallelogram?                      What are the properties of a rhombus?                      What are the properties of a trapezoid (U.S.) or trapezium (UK)?</p>
<p><b>Conceptual Questions</b>                  Why or How?</p>	<p>Why can a square also be a rectangle?                      How is a rhombus related to a parallelogram?</p>
<p><b>Debatable/Provocative Question</b></p>	<p>Do planes (two-dimensional shapes, not the flying type!) exist in the real world?</p>

FIGURE M3.3: TEMPLATE FOR WRITING GUIDING QUESTIONS

<b>Guiding Questions</b>		<b>Unit of Study:</b>
<b>Generalization:</b>		
<b>Factual Questions</b> What?		
<b>Conceptual Questions</b> Why or How?		
<b>Debatable/Provocative Question</b>		



**Unit of Study:**

**Generalization:**

**Factual Questions**

**What?**

**Conceptual Questions**

**Why or How?**

**Debatable/Provocative Question**

FIGURE M3.4: EXAMPLE OF KUDS FOR GEOMETRY GRADE 8/YEAR 7

Unit of Study: Geometry		
Know	Understand (Generalizations)	Do
<p>A pair of perpendicular number lines, called axes, define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates.</p> <p>The properties of two-dimensional figures</p>	<p>Ordered pairs define the relationship between values on a coordinate plane that describe real-life situations.</p> <p>Properties of two-dimensional plane shapes allow mathematicians to classify and compare common attributes.</p>	<p>Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.</p>

FIGURE M3.5: TEMPLATE FOR WRITING KUDS

<b>Unit of Study:</b>		
Know	Understand (Generalizations)	Do

## Discussion Questions for Module 3

1. Why should you include KUDs in your unit planning?
2. Where do generalizations fit in your KUDs?
3. What are the advantages of using guiding questions when planning a unit of work?
4. How will you use the three types of guiding questions in your practice?
5. How will you use the provocative/debatable questions?


## An Opportunity for Reflection

Write down three do's and three don'ts based on what you have learned about guiding questions in this module.

### Three Do's



### Three Don'ts



# Math Unit Planning

## What Does Concept-Based Unit Planning Look Like?

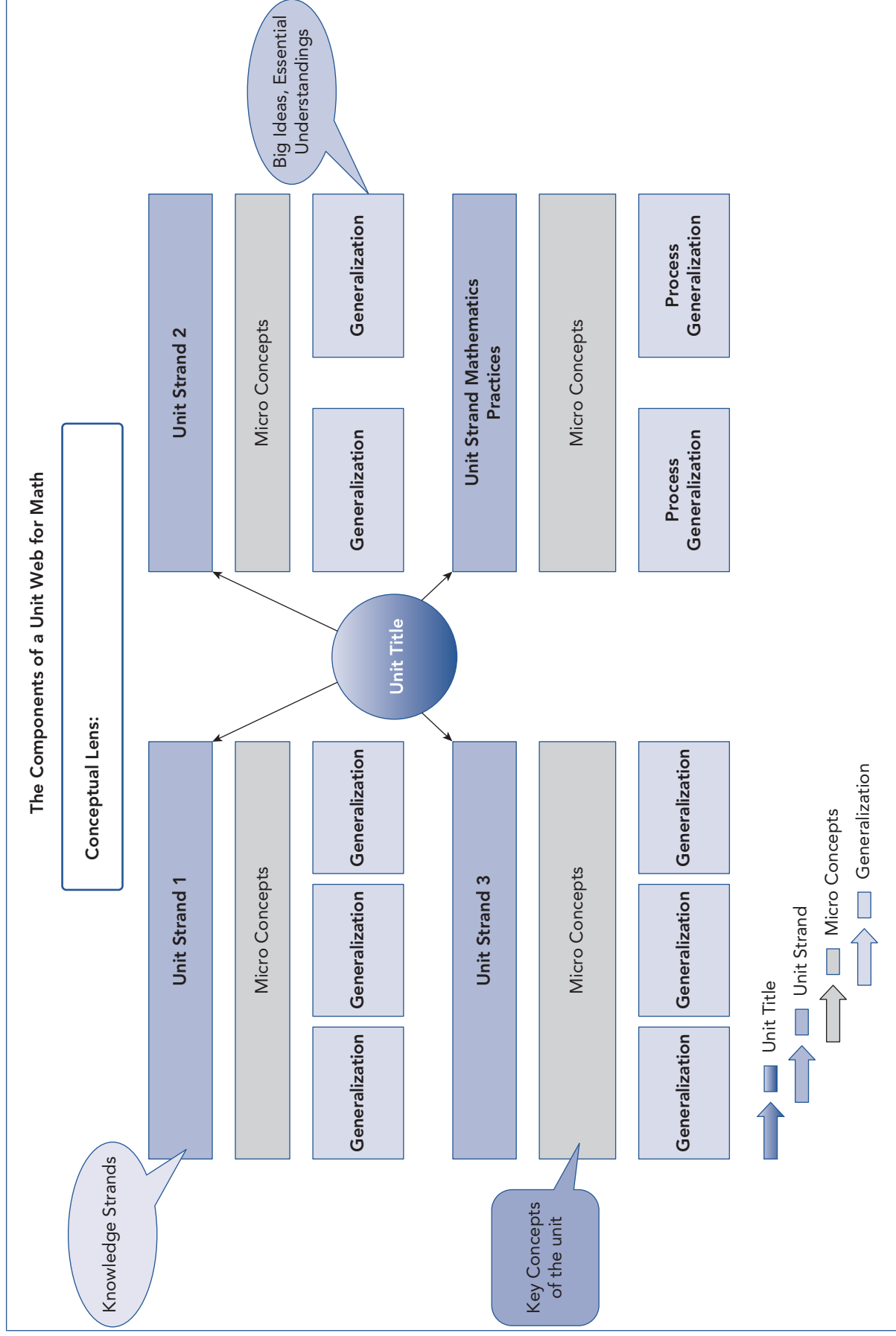
### Main Text: Chapter 4. How Do I Plan Units of Work for a Concept-Based Curriculum?

Time should be invested in planning a unit of work to ensure that students are given opportunities to draw the essential understandings from it. A unit web is an essential tool for planning a concept-based unit of work. A unit web contains critical content topics and concepts and gives an overview of the depth and breadth of the unit of instruction. Erickson has said the more complete the planning web, the stronger the conceptual understanding. When planning a unit of work for mathematics, it is important to include a strand titled “Mathematical Practices,” which represents the process generalizations. The other strands around the unit web represent the knowledge generalizations.

In this module you will find the following resources:

- A generic math unit web template
- A template for a concept-based unit planner
- A template for a concept-based weekly planner
- Discussion questions for Module 4
- An opportunity for reflection

FIGURE M4.1: GENERIC MATHEMATICS UNIT WEB TEMPLATE



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FIGURE M4.2 TEMPLATE FOR CONCEPT-BASED UNIT PLANNER

<b>Unit Title:</b>	<b>Conceptual Lens:</b>	<b>Time: Allocation:</b>	<b>Grade/Year:</b>
Unit Overview:	Concepts in the Unit:		
We would like students to <b>know</b> . . .	We would like students to <b>understand</b> that . . .	We would like students to be able to <b>do</b> . . .	

Generalization	Guiding Questions
	<b>Factual:</b>  <b>Conceptual:</b>
	<b>Factual:</b>  <b>Conceptual:</b>

(Continued)

FIGURE M4.2 (CONTINUED)

Generalization	Guiding Questions
	<p><b>Factual:</b></p> <p><b>Conceptual:</b></p>
	<p><b>Factual:</b></p> <p><b>Conceptual:</b></p>
	<p><b>Factual:</b></p> <p><b>Conceptual:</b></p>
<b>Debatable Unit Questions</b>	



FIGURE M4.3 TEMPLATE FOR CONCEPT-BASED WEEKLY PLANNER

Weekly Planner				
Week	Concepts	Generalization	Learning Experience	Differentiation Strategies

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## Discussion Questions for Module 4

1. What is the purpose of a unit web?
2. What is the purpose of a unit planner?
3. Why do we need to include KUDs (know, understand, and do) when planning units of work?
4. What is the purpose of a weekly planner?
5. How will you develop unit planners in the future?
6. How will you resource your units with concept-based learning experiences?

## An Opportunity for Reflection

**I was surprised . . .**

**I learned . . .**

**I feel . . .**

# How Do I Design Inductive, Inquiry-Based Math Tasks?

## Main Text: Chapter 5. How Do I Captivate Students? Eight Strategies for Engaging the Hearts and Minds of Students

One of the tenets of concept-based curriculum and instruction is the use of inductive teaching approaches. This means students are given specific numerical examples to work out and are guided to the generalizations. It may be appropriate to use different levels of inquiry—structured, guided, or open—depending on teacher experience and student readiness. The levels of inquiry also provide a differentiation strategy.

In this module you will find the following resources:

- Examples of different levels of inquiry using inductive approaches
- An example of an inductive inquiry task and the use of a hint jar
- A template for planning inductive inquiry tasks
- Discussion questions for Module 5
- An opportunity for reflection

FIGURE M5.1: EXAMPLE OF STRUCTURED INQUIRY TASK: LINEAR EQUATIONS

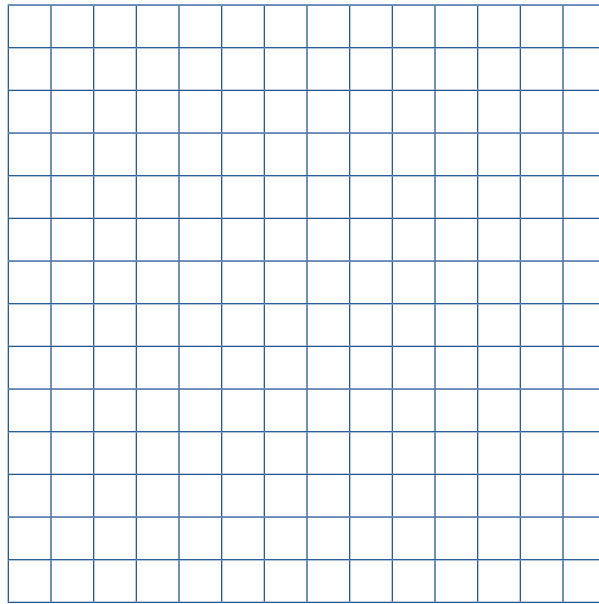
# Investigating Straight Lines

## Part 1

The cost of a taxi is a flat rate of \$1 and then \$3 for every mile travelled. Complete this table in which  $y$  represents the total cost of the taxi ride and  $x$  represents miles travelled.

$x$ (miles)	0	1	2	3	4	5	6	7	8
$y$ (total fare)									

Draw the  $y$  and  $x$  axes below and plot the points from the table.



Can you think of a function that represents the total fare for a taxi ride travelling  $x$  miles?

$y =$
-------

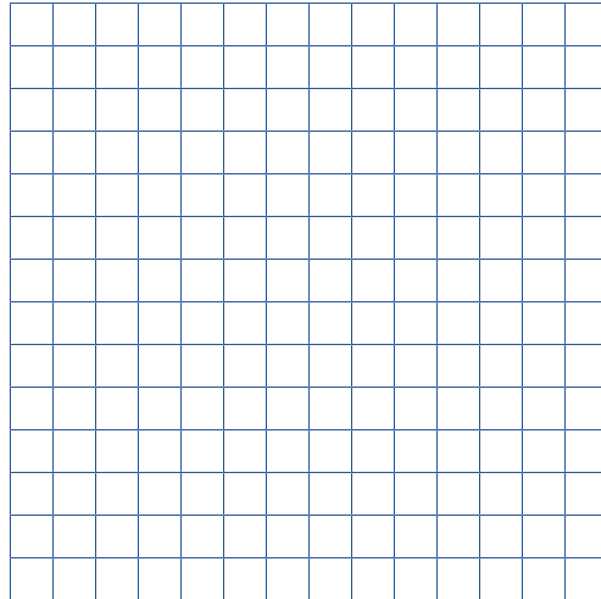
What is the cost per mile? How is this represented on the graph?

**Part 2**

The cost of a taxi is a flat rate of \$2 and then \$4 for every mile travelled. Complete this table in which  $y$  represents the total cost of the taxi ride and  $x$  represents miles travelled.

$x$ (miles)	0	1	2	3	4	5	6	7	8
$y$ (total fare)									

Draw the  $y$  and  $x$  axes below and plot the points from the table.



Can you think of a function that represents the total fare for a taxi ride travelling  $x$  miles?

$y =$
-------

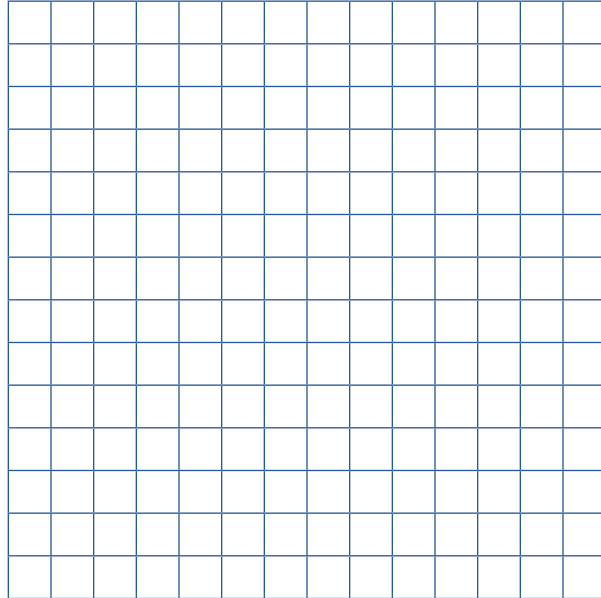
What is the cost per mile? How is this represented on the graph?

**Part 3**

The cost of a taxi is a flat rate of \$3 and then \$9 for every mile travelled. Complete this table in which  $y$  represents the total cost of the taxi ride and  $x$  represents miles travelled.

$x$ (miles)	0	1	2	3	4	5	6	7	8
$y$ (total fare)									

Draw the  $y$  and  $x$  axes below and plot the points from the table.



Can you think of a function that represents the total fare for a taxi ride travelling  $x$  miles?

$y =$

What is the cost per mile? How is this represented on the graph?

Can you think of a function that represents the total fare for a taxi ride travelling  $x$  miles if the flat rate was  $m$  and the cost per mile was  $b$ ?

$y =$

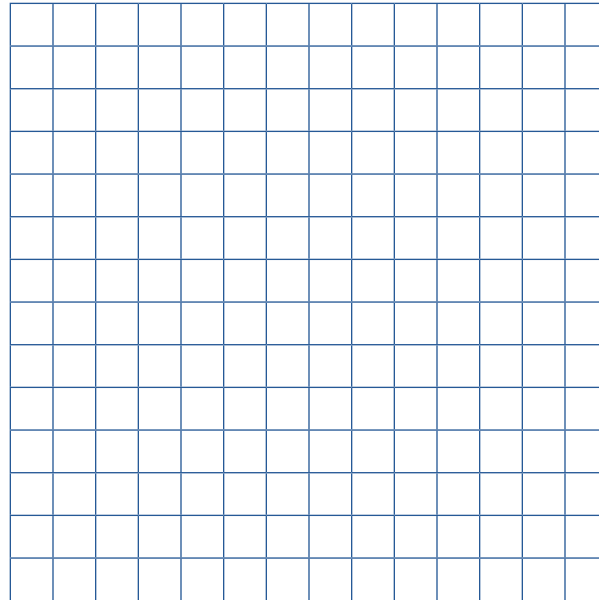
What is the cost per mile? How is this represented on the graph?

# Investigating Straight Lines

## Part 1

The cost of a taxi is a flat rate of \$1 and then \$3 for every mile travelled.

Draw the  $y$  and  $x$  axes below and plot the points that represent the total cost ( $y$ ) of a taxi ride for different miles ( $x$ ) travelled.



Can you think of a function that represents the total fare for a taxi ride travelling  $x$  miles?

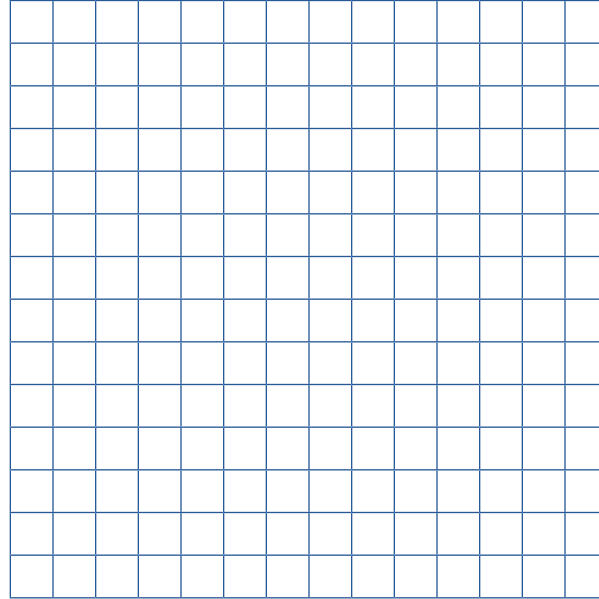
$y =$
-------

What is the cost per mile? How is this represented on the graph?

**Part 2**

The cost of a taxi is a flat rate of \$2 and then \$4 for every mile travelled.

Draw the y and x axes below and plot the points that represent the total cost (y) of a taxi ride for different miles (x) travelled.



Can you think of a function that represents the total fare for a taxi ride travelling x miles?

$$y =$$

What is the cost per mile? How is this represented on the graph?

Can you think of a function that represents the total fare for a taxi ride travelling x miles if the flat rate was  $m$  and the cost per mile was  $b$ ?

$$y =$$

What is the cost per mile? How is this represented on the graph?



FIGURE M5.3: EXAMPLE OF OPEN INQUIRY TASK: LINEAR EQUATIONS

## Investigating Straight Lines

Investigate the effects of the parameters  $m$  and  $b$  on the linear function  $y = mx + b$  and explain what happens when  $m$  and  $b$  take different values. Use real-life examples to illustrate your explanations.

FIGURE M5.4: EXAMPLE OF INDUCTIVE INQUIRY TASK: ADDING AND SUBTRACTING NEGATIVE NUMBERS

## Temperature Scales

The lowest temperature recorded in Washington is  $-26^{\circ}\text{C}$ .

Mark this on the temperature scale on the right with a **W**.

A heat wave sweeps Washington and takes away this cold temperature, resulting in a temperature of  $0^{\circ}\text{C}$ .

This could be represented as

$-26$	take away	$-26$
Or in mathematical symbols		
$-26 - (-26) =$		
Another way to express this heat wave is to add 26		
$-26 + 26 =$		
What conclusion can you make about subtracting a negative number? Provide an example of your own that shows how you take away something that is already negative.		



FIGURE M5.5: EXAMPLE OF INDUCTIVE INQUIRY TASK: USING THE TRI-MIND ACTIVITY

## Hint Jar

You owe your mum \$10. This means you have  $-\$10$ . How do you take this away? Explain and show the sum.

You tell someone either to eat, to not eat, or to **NOT** not eat. What does **NOT** not eat mean?

Good things happen to good people = GOOD (positive)

Good things happen to bad people = BAD (negative)

Bad things happen to good people =

Bad things happen to bad people =

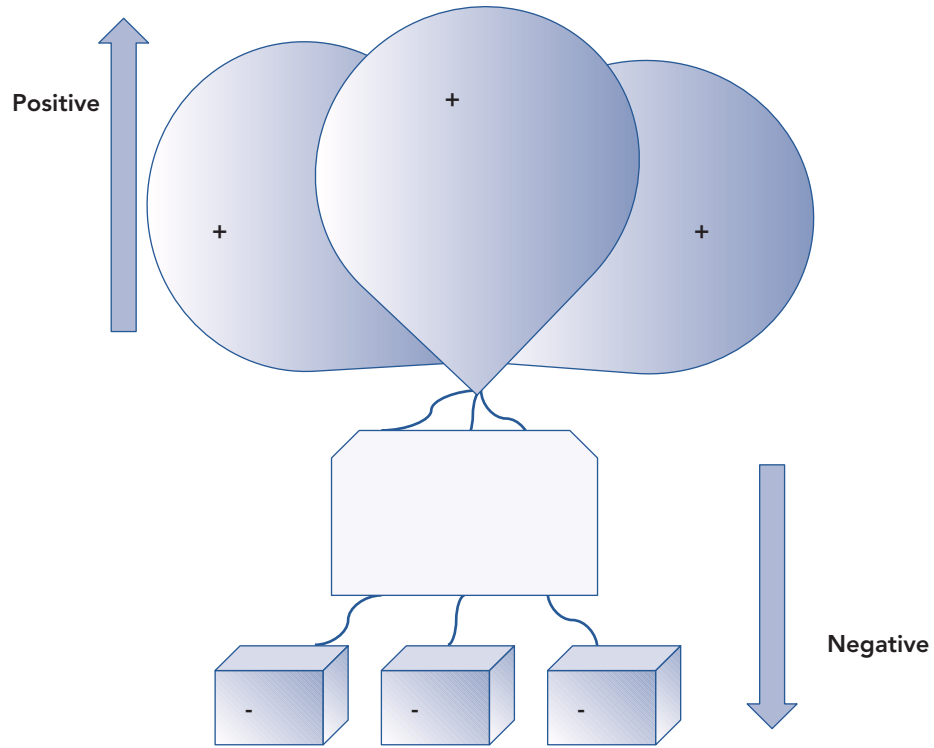
Think of your own analogy and write it here:

FIGURE M5.6: EXAMPLE OF INDUCTIVE INQUIRY TASK: ADDING AND SUBTRACTING POSITIVE AND NEGATIVE NUMBERS

## Hot Air Balloons

The hot air balloon basket is floating in the sky with the balloons and weights. Fill in the following table:

Take Away or Add Balloons	Basket Moves up (+) or Down (-)
Add 3 balloons	
Take away 3 balloons	
Take Away or Add Weights	Basket Moves up (+) or Down (-)
Add -3 weights	
Take away -3 weights	
Write your own sums that represent the four operations above:	



What generalizations can you make about adding and subtracting positive and negative numbers? Provide several examples to illustrate your explanations.

FIGURE M5.7: TEMPLATE FOR DESIGNING INDUCTIVE INQUIRY TASKS

### Designing Inductive Inquiry Tasks

<b>Generalization:</b>	
------------------------	--

### Guiding questions

Factual Questions	
Conceptual Questions	
Debatable Questions	

Inductive Student Task	Prompts
Specific numerical example	
Second specific numerical example	
Third specific numerical example, if appropriate	
Students form generalizations	

## Discussion Questions for Module 5

1. What is the difference between inductive and deductive teaching approaches?
2. What are the different levels of inquiry?
3. When and how would you use the different levels of inquiry?
4. How do you use the hint jar for the activity on negative numbers?

## An Opportunity for Reflection

**Write a headline about inductive inquiry tasks**

This thinking routine helps you to summarize the essence or the most important ideas to you regarding inductive inquiry tasks.

# Designing Differentiated Tasks

## How Do I Plan Tasks to Ensure Every Student Learns?

**Main Text: Chapter 5. How Do I Captivate Students?  
Eight Strategies for Engaging the Hearts and Minds of Students, and  
Chapter 6. How Do I Know My Students Understand the Concepts?  
Assessment Strategies**

Generally, ongoing formative assessment is employed throughout a unit of work. This is often not graded as it provides feedback on how your students are progressing in their learning and informs future instruction. Diagnostic assessment allows you to identify the starting point for individual students, to facilitate differentiation of instruction. Summative assessment allows you to evaluate learning at the end of a unit, which may be used for future teaching and learning plans. The suggested assessments may be used for diagnostic, formative, or summative assessment. For example, I use Frayer's model as a diagnostic at the beginning of a unit to assess students' preexisting knowledge; I also use it during a unit to check for understanding and as a summative assessment to give students opportunities to show me what they know. Included in this chapter are differentiation strategies such as the tri-Mind thinking styles and hint cards.

In this module you will find the following resources:

- Examples of pre-assessment strategies and formative and summative assessments
- Examples of tri-mind tasks for differentiating thinking styles
- A template for designing tri-mind tasks
- Discussion questions for Module 6
- An opportunity for reflection



FIGURE M6.1: EXAMPLE OF A PRE-ASSESSMENT MATCHING ACTIVITY USING VOCABULARY WORDS

## Matching

Match the word with the correct definition.

<b>rhombus/diamond</b>	<b>four-sided 2D shape</b>
<b>rectangle</b>	<b>four-sided 2D shape, opposite sides parallel, opposite sides equal</b>
<b>square</b>	<b>four-sided 2D shape, pair of opposite sides parallel</b>
<b>parallelogram</b>	<b>four-sided 2D shape, equal sides, 4 right angles, opposite sides parallel</b>
<b>trapezoid (U.S.) or trapezium (UK)</b>	<b>four-sided 2D shape, all sides equal length, opposite sides parallel, opposite angles equal</b>
<b>quadrilateral</b>	<b>four-sided 2D shape, opposite sides parallel, opposite sides equal in length, 4 right angles</b>
<b>kite</b>	<b>adjacent sides equal in length</b>

FIGURE M6.2: EXAMPLE OF A PRE-ASSESSMENT MATCHING ACTIVITY USING VISUAL CUES

# Matching

Match the vocabulary words with the pictures.



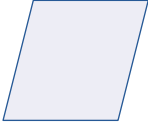


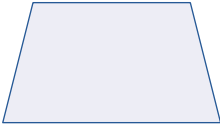
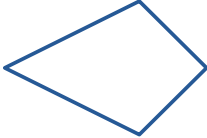
<b>rhombus/diamond</b>	
<b>rectangle</b>	
<b>square</b>	
<b>parallelogram</b>	
<b>trapezoid (U.S.)/trapezium (UK)</b>	
<b>quadrilateral</b>	
<b>kite</b>	

FIGURE M6.3: EXAMPLES OF QUICK QUESTIONS FOR PRE-ASSESSMENT

### Quick Questions on Mini Whiteboards

Starting off with quick questions informs you which concepts students already know and which they do not. Students hold up their answers on mini whiteboards so you can quickly and easily identify which students already know these concepts. Different stations could be set up in the classroom after this pre-assessment to address these ideas. Students who already know these concepts well can move on to line symmetry.

**What is an acute angle?**

**What is an obtuse angle?**

**What is a reflex angle?**

**What is a right angle?**

**Draw a set of parallel lines and include an explanation.**

**Draw a set of perpendicular lines and include an explanation.**

FIGURE M6.4: EXAMPLE OF USING A CONCEPT MAP FOR PRE-ASSESSMENT

## Concept Maps

In pairs, write down what you know about angles.

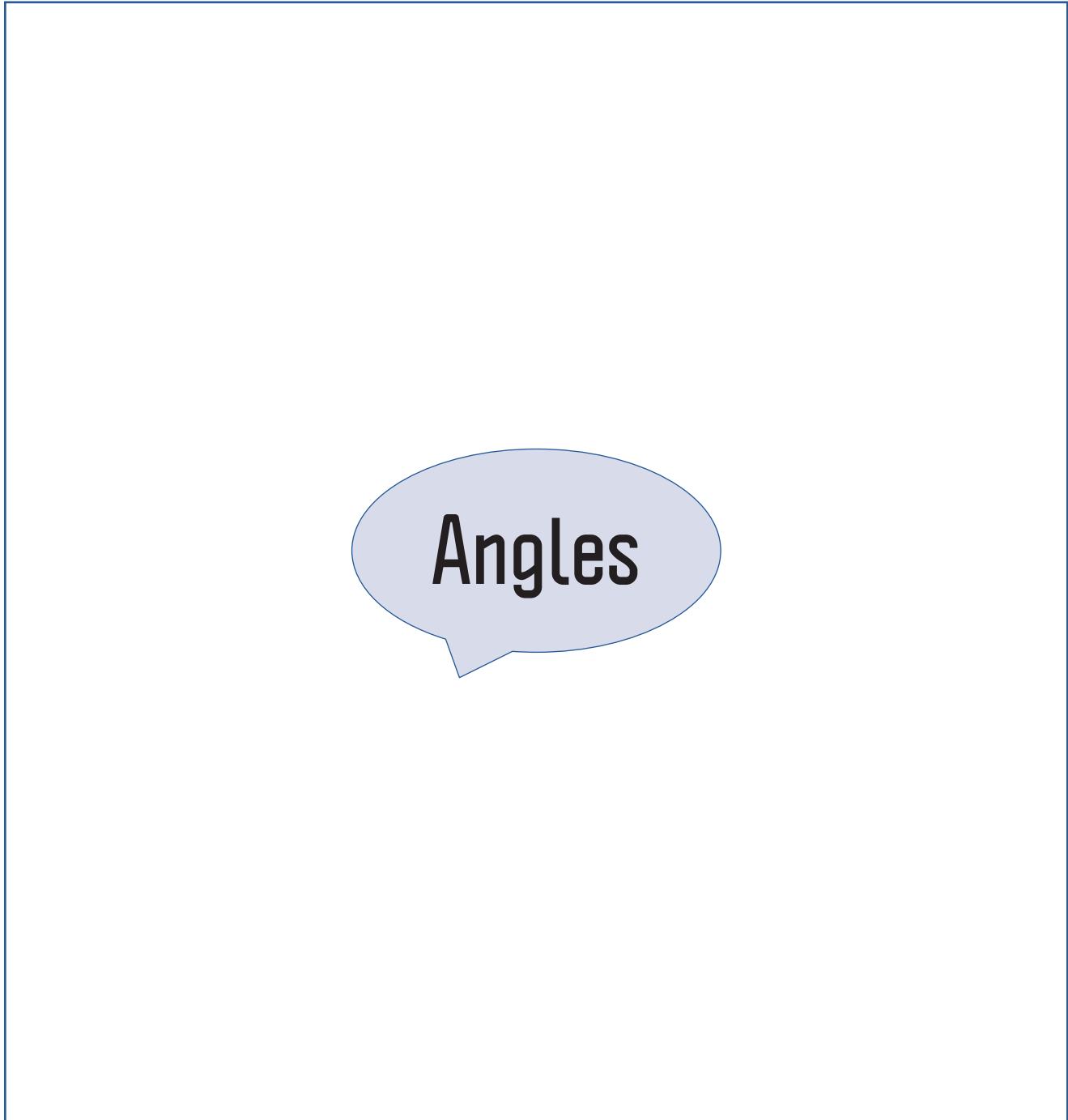


FIGURE M6.5: EXAMPLE USING FRAVER'S MODEL FOR ANGLES

## Frayer's Model

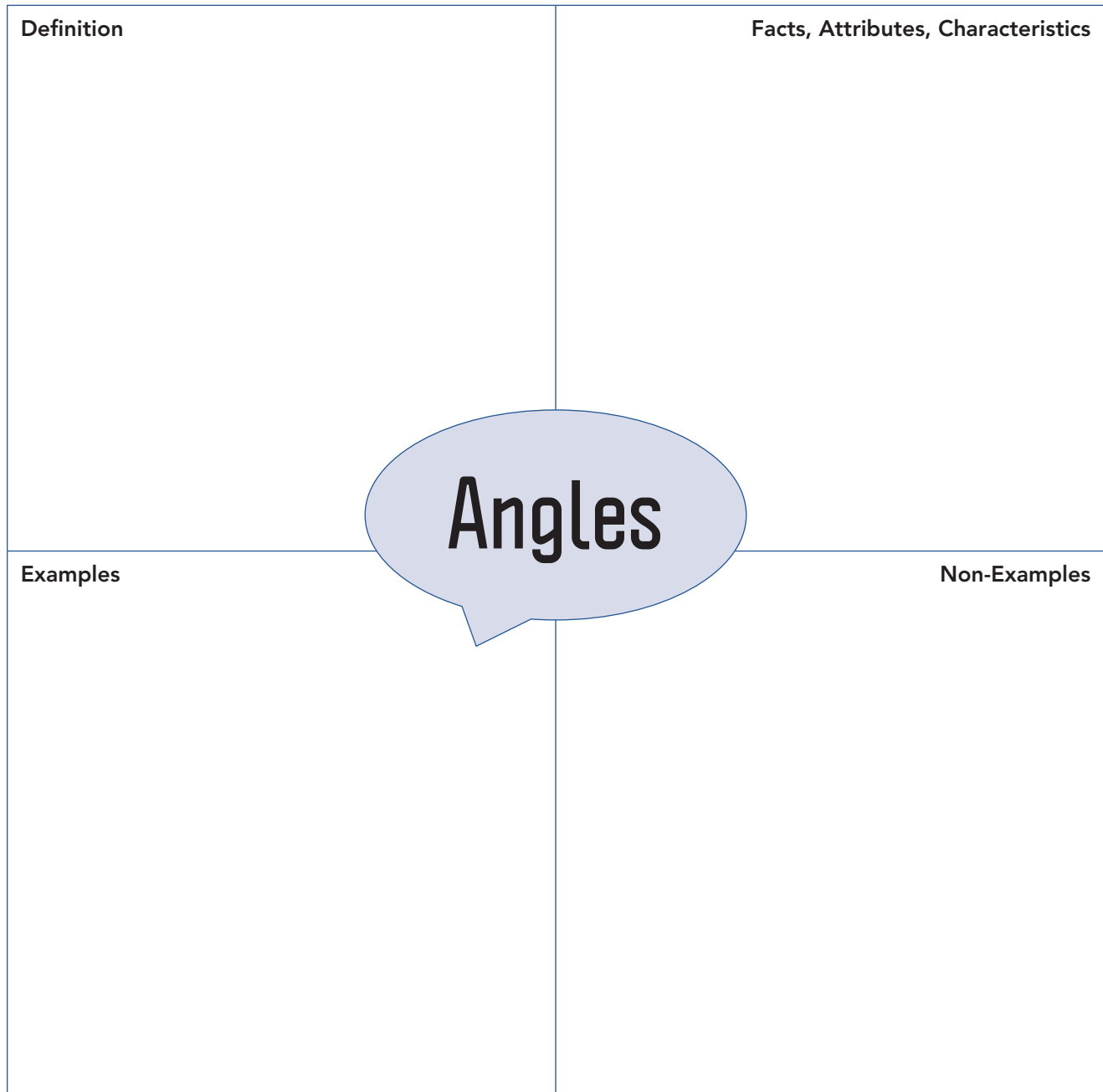


FIGURE M6.6: EXAMPLE OF USING WINDOW PANES FOR ASSESSMENT (DIAGNOSTIC, FORMATIVE, OR SUMMATIVE) OF TRIGONOMETRY

## Window Panes

Write down what you know about trigonometry.

Trigonometry	
<b>Right-angled trigonometry</b>	<b>Three-dimensional trigonometry</b>
<b>Non-right-angled trigonometry</b>	<b>Explain how right-angled and non-right-angled trigonometry are connected to three-dimensional trigonometry.</b>

You can also use window panes by including your KUDs (what we want our students to Know, Understand, and Do) and asking some questions here for pre-assessment.

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FIGURE M6.7: EXAMPLE OF A TRI-MIND ACTIVITY FOR MEASUREMENT

## Measurement

1. Choose one of the three options below.
2. Find two other people who have also made the same choice as you.
3. Work on your task together and be ready to share.

Make a summary and explain all the key concepts in the topic of measurement.	Explain how you would use all the concepts of measurement in real life.	Create a graphic (visual) or skit of how all the concepts of measurement are related. Provide examples to illustrate your points.
<b>My choice is:</b>	<b>I chose this because:</b>	
<b>The members of my group are:</b>	<b>Outline of individual roles and action plan:</b>	

FIGURE M6.8: EXPLANATION OF TRI-MIND ACTIVITY

## Topic for Tri-Mind:

1. Choose one of the options below.
2. Find two other people who have also made the same choice as you.
3. Work on your task together and be ready to share.

<b>Analytical Thinkers</b> <i>Example: Write a definition of the topic.</i>	<b>Practical Thinkers</b> <i>Example: Show the application of the topic. How would you use this?</i>	<b>Creative Thinkers</b> <i>Example: Create a metaphor, visual, or skit.</i>
<p><b>My choice is:</b></p>	<p><b>I chose this because:</b></p> <p><b>Students may recognize their preferred style; however, it is also important to encourage student to try different approaches and different thinking styles.</b></p>	
<p><b>The members of my group are:</b></p>	<p><b>Outline of individual roles and action plan:</b></p> <p><b>In any group work, it is important that each member has a clearly defined role so all students are activity involved.</b></p>	



## Discussion Questions for Module 6

1. What is the purpose of pre-assessment?
2. What are other differentiation strategies for the math classroom?
3. How would you use formative assessment strategies in your own classroom?
4. What are some other ways you can use tri-mind activities in your classroom?

## An Opportunity for Reflection

### Think, Puzzle, Explore

1. What do you **think** you know about differentiation?
2. What questions or **puzzles** do you have about differentiation?
3. How can you **explore** differentiation further to support your own classroom practice?

# Designing Math Performance Tasks

## How Do I Engage Students With Authentic Performance Tasks?

**Main Text: Chapter 6. How Do I Know My Students Understand the Concepts? Assessment Strategies, and Chapter 7. How Do I Integrate Technology to Foster Conceptual Understanding?**

Performance tasks allow students to demonstrate what they need to *know*, *understand*, and *do* from a unit of work. Lynn Erickson has developed the “What, Why, and How” guide to support the process of designing a performance task. To develop a performance of deep understanding, Erickson (2007) encourages teachers to “pick up exact language from the generalization (the Why) and build that language into the task to ensure the tie between the task and the conceptual understanding.”

RAFTS (Holston & Santa, 1985) is another tool for writing performance tasks, which helps to identify role, audience, format, topic, and the use of strong verbs and adjectives, all put together in an engaging scenario. Many lesson plans have students use skills to learn factual content and do not extend student thinking to the conceptual level. These are referred to as “activities.” Look at the following examples and determine which is a performance task of deep understanding and which is a lower level activity.

FIGURE M7.1: PERFORMANCE TASK VS. ACTIVITY

Performance Task or Activity?	
Use the ratios of sine, cosine, and tangent to find the unknown angles and unknown sides of the following right-angled triangles.	Island School was built in its current location in 1973 and the buildings, while well maintained, need to be rebuilt. The buildings are literally falling apart! The architects would like to preserve the playgrounds that all the buildings are centered around and need to know the heights of all seven blocks for the new plan. Find out the heights of all the blocks at Island School and make a scale drawing of the current school. This scale drawing should be a bird's-eye view and include all dimensions including the height. Design your own school ensuring that you accommodate all faculties and facilities listed below. You may use any format to present your findings.

In this module you will find the following resources:

- Templates for designing a math performance task
- Examples of task formats and roles
- Discussion questions for Module 7
- An opportunity for reflection

FIGURE M7.2: TEMPLATE FOR DESIGNING A MATH PERFORMANCE TASK USING WHAT, WHY, AND HOW

Unit Title/Key Topic:

What	Investigate . . . (unit title or key topic)
Why	In order to understand that . . . (generalization)
How	Performance task/engaging scenario for students

Adapted from Lynn Erickson (2007).

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FIGURE M7.3: TEMPLATE FOR DESIGNING A MATH PERFORMANCE TASK USING RAFTS (ROLE, AUDIENCE, FORMAT, TOPIC, AND STRONG VERBS AND ADJECTIVES)

Unit Title/Key Topic:

<b>R = Role</b>	
<b>A = Audience</b>	
<b>F = Format</b>	
<b>T = Topic</b>	
<b>S = Strong Verbs and Adjectives</b>	

Adapted from Holston and Santa (1985).

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FIGURE M7.4: EXAMPLES OF TASK FORMATS AND TASK ROLES

Task Formats		
Lab report	Movie/Documentary	Panel discussion
Newsletter	Movie review	Experiment
Cartoon	Debate	Diorama & explanation
Advertisement	Action plan	Applet using software
Jingle/Song	Invention	Museum exhibit
Play/Skit	Journal entry	Sculpture or drawing
Letter	Critique	Musical story
Top tips	Story board	Police report
Animation	3D model	Board game
Short story	Proposal	Map
Website	Travel brochure	Biography

Task Roles		
Scientist, doctor	Movie maker	Panel discussion
Secretary	Movie reviewer	Scientist
Cartoonist	Debater	Set designer
Advertiser	Leader, hero	Software designer
Musical ad maker	Inventor	Museum curator
Playwright	Journal writer	Artist
Writer	Reviewer	Musician
Friend, citizen	News presenter	Policeperson
Expert in a field/Teacher	3D model maker	Game maker
Storyteller	Proposal writer	Cartographer
Web page designer	Travel agent, tour guide	Biographer

## Discussion Questions for Module 7

1. What is the purpose of a task planner?
2. Why is it important to design a task with an engaging scenario?
3. How can you plan for differentiation when designing a performance task?
4. What is the difference between a performance task and an activity?
5. Why is it critical to reflect the exact language of the generalization in the actual task?
6. Can you think of other formats and roles not listed in Figure M7.4?

## An Opportunity for Reflection

**P**

**Positive:**

**M**

**Minus:**

**I**

**Interesting:**

# Developing the Concept-Based Teacher

## How Do I Develop as a Teacher to Integrate Concept-Based Curriculum in My Planning and Instruction?

### Main Text: Chapter 8. What Do Ideal Concept-Based Math Classrooms Look Like?

The journey to developing as a concept-based teacher will require a commitment to fostering and provoking deep conceptual understanding in our students. Lois Lanning (Erickson & Lanning, 2014) has developed rubrics to help the developing concept-based teacher identify progress in terms of lesson planning, instruction, and the understanding of the concept-based model overall (Figures M8.1–M8.3).

Identify which level you are at now for each of the rubric components in concept-based curriculum and instruction and use the target-setting templates (Figures M8.4–M8.6) to reflect on how you are going to make personal progress as a concept-based teacher. Use the template in Figure M8.7 to draft your personal action plan.

Once you have created your personal action plan, set targets as a faculty using the SMART goals faculty development plan in Figure M8.8. The SMART goals template can also be used schoolwide. The use of SMART goals will ensure your targets are specific, measurable, achievable/assignable, realistic, and time bound.

In this module you will find the following resources:

- A rubric for concept-based instruction
- A rubric for concept-based lesson planning
- A rubric for understanding concept-based curriculum and instruction
- Target-setting templates for each of the rubrics: concept-based instruction, concept-based lesson planning, and understanding concept-based curriculum and instruction
- A personal action plan template
- A SMART faculty development plan template
- Discussion questions for Module 8
- An opportunity for reflection



FIGURE M8.1: RUBRIC: CONCEPT-BASED INSTRUCTION BY DR. LOIS A. LANNING

	Novice	Emerging	Master
<b>Lesson Opening</b>	<ul style="list-style-type: none"> <li>Lesson opens by launching directly into an activity without providing an overview or clear directions</li> <li>The lesson's target generalization is posted or stated at the beginning of the lesson rather than drawn from students throughout</li> <li>The lesson opening is accurate but bland and leaves students disinterested as it is more of a teacher's monologue</li> </ul>	<ul style="list-style-type: none"> <li>The lesson opening sets the stage for inductive teaching that will draw out conceptual understandings from the students (e.g., examples are posted, intriguing questions presented, an interesting scenario shared, relevant concepts posed) but the opening is overly detailed and too long</li> </ul>	<ul style="list-style-type: none"> <li>The opening clearly communicates an engaging, captivating overview of the lesson that connects and extends previous learning</li> </ul>
<b>During</b> Instruction reflects modeling, facilitating, and mediating conceptual understanding	<ul style="list-style-type: none"> <li>The lesson follows the written lesson plan but since the plan does not address all the elements of an effective concept-based lesson, instruction falls short</li> <li>Concepts are somewhat apparent in the lesson with little attention to how to use concepts to create intellectual engagement and deepen students' understanding</li> <li>Focus is less on the transfer of learning and more on task completion</li> </ul>	<ul style="list-style-type: none"> <li>Follows the concept-based lesson plan prescriptively</li> <li>Maintain the emotional engagement of students by using examples and resources</li> <li>There is some gradual releases of responsibility of learning from teacher to student, but the teacher is assuming most of the cognitive work</li> </ul>	<ul style="list-style-type: none"> <li>The execution of the lesson plan is well paced, while being flexible and responsive to anticipated student needs (and needs that arise throughout the lesson)</li> <li>There is a conscious and consistent development of students' synergistic thinking through instructional techniques and thought provoking examples and through learning experiences and resources that bridge to a deeper, conceptual idea (understanding)</li> </ul>

	Novice	Emerging	Master
	<ul style="list-style-type: none"> <li>• Instruction employs different kinds of questions as the main tool for encouraging the transfer of learning but still over-relies on factual questions</li> <li>• Instruction remains predominantly teacher centered</li> <li>• Student participation is predominantly in response to teacher questioning and evaluation</li> </ul>	<ul style="list-style-type: none"> <li>• Most student's are engaged in the learning while clusters of students may remain off task or disinterested due to ineffective level of challenge or lack of relevance</li> </ul>	<ul style="list-style-type: none"> <li>• Teaching uses a variety of techniques to support the transfer of learning and deepening of understanding (e.g. questions, asking for other examples/non-examples of the same concept or generalization, feedback, and by asking students for an analysis of their reasoning with supporting evidence)</li> <li>• Gradual release of responsibility and ownership for learning from teacher to student is clear</li> <li>• Continuous monitoring of students' independent and collaborative group work with timely relevant feedback and questions that facilitate and mediate the learning process</li> </ul>
<b>Lesson Closing</b>	<ul style="list-style-type: none"> <li>• The teacher recaps the learning experiences in the lesson</li> </ul>	<ul style="list-style-type: none"> <li>• There is closing assessment (formative or summative) of the knowledge and skills students learned and an attempt to determine students' level of conceptual understanding</li> <li>• Relevant practice beyond the lesson is assigned</li> </ul>	<ul style="list-style-type: none"> <li>• Evidence of learning (formative or summative) the lesson's targeted knowledge, skills, and understanding is collected</li> <li>• Collaboratively the teacher and students reflect on and analyze the success of the learning (process and product)</li> <li>• Students learn how the learning will build future learning targets</li> </ul>

SOURCE: *Transitioning to Concept-Based Curriculum and Instruction*, Corwin Press Publishers, Thousand Oaks, CA

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FIGURE M8.2. THE DEVELOPING CONCEPT-BASED TEACHER: CONCEPT-BASED LESSON PLANNING

Components of Lesson Plan		Novice	Emerging	Master
<p><b>Lesson opening:</b> An explicit and engaging summary of the work to be accomplished that triggers synergistic thinking</p> <p><b>Learning targets:</b> What students are expected to know (factual knowledge), understand (generalization), and be able to do (skills)</p> <p><b>Guiding questions:</b> The three different types of questions (factual, conceptual, provocative) serve as a bridging tool for conceptual thinking and problem solving.</p> <p><b>Learning experiences:</b> Intellectually engaging student work that provides opportunities for students to practice their learning and to arrive at the target generalization (conceptual understanding)</p>	<p>Lesson opens by stating the activities students will experience in the lesson and may include stating the generalization the lesson will be teaching toward</p> <p>What students must Know and/or be able to Do) is listed in the lesson plan</p> <p>Questions in the lesson plan focus heavily on factual knowledge and routine skills</p>	<p>Lesson opening contains a conceptual lens but the weak tie to content fails to engage synergistic thinking</p> <p>Targets identify what students must Know, Understand (generalization), and Do, but there may be more learning targets than can be accomplished in-depth within the lesson timeframe</p> <p>Lesson questions reflect different types (factual, conceptual, and possibly provocative questions) and anticipate student misconceptions</p> <p>The student work attempts to pursue conceptual understandings but may not provide a clear pathway with enough examples or scaffolds for students to realize the conceptual understanding (generalization)</p> <p>Lesson plan shows efforts to design student work to engage students' interest and offers some student choice</p>	<p>The lesson opening engages synergistic thinking by asking students to consider the knowledge and/or skills they will be learning through a conceptual question(s) or lens</p> <p>Learning targets represent what students are expected to Know, Understand, and Do; the limited number of learning targets allows for in-depth, focused instruction and learning</p> <p>Potential questions are of different types (factual, conceptual, provocative) and are listed throughout the plan</p> <p>The lesson plan shows a deliberate effort to use questions to help students bridge from the factual to the conceptual level of understanding</p> <p>The student work requires students to cognitively wrestle with and synthesize the knowledge, skills, and concepts under study in relevant contexts that lead to the realization of the generalization</p>	

Components of Lesson Plan	Novice	Emerging	Master
<p><b>Assessment methods:</b> Assessment types are selected according to the lesson's learning targets (know, understand, do) and to the assessment purposes (formative &amp; summative) in order to capture evidence of students' learning (process &amp; product) which will then inform instruction.</p> <p><b>Differentiation:</b> Lesson adjustments are planned, as needed, for the content students are expected to master, the process students will use to access the content, and the product students will produce to show their learning. The conceptual understanding (generalization) all students are expected to realize remains consistent for all students.</p>	<p>The targeted knowledge and skills are indicated in the plan, but the learning experiences do not require students to apply their learning in relevant contexts, which would clearly lead to conceptual understanding and transfer across learning situations</p> <p>The student work required in the lesson primarily relies on worksheets, disconnected skills, and facts that are not authentic or intellectually/emotionally engaging for students</p>	<p>Assignments are intellectually and emotionally engaging but are not at the appropriate level of challenge for all students</p> <p>Assessment types are varied and help monitor students developing knowledge and skills</p> <p>Assessments of understanding, aligned to the target generalization, are not clear</p>	<p>Student work is at the appropriate level of challenge, is intellectually and emotionally engaging, meaningful, and relevant to the discipline, and provides appropriate student choice</p> <p>The learning experiences are deliberately designed to enhance the transfer of learning across other disciplines and situations</p> <p>Assessment types are varied so they assess students' developing knowledge, skills, and understandings (generalization) and allow for timely feedback</p> <p>Assessments provide relevant information about students' process of learning as well as their learning products</p> <p>Student self-assessment is valued</p> <p>Plans for differentiation to meet the needs of all learners are included and support all students meeting a common conceptual understanding (generalization)</p>
<p><b>Lesson design:</b> In a deductive lesson design, the teacher states the learning targets (including generalization) to the learners at the beginning of the instruction. In an inductive design, students construct their understandings through an inquiry process.</p> <p><b>Closing:</b> Plan for a way that evidence of learning can be reviewed collectively</p>	<p>Assessment types are limited so it is difficult to know the degree of students' learning and progress toward conceptual understanding</p> <p>Plans for differentiation may be stated but lack relevance to individual student learning needs</p> <p>The lesson design is deductive (e.g., objective to example vs. example to generalization)</p>	<p>Plans for differentiation are included for students who need support (e.g., special education, ELL) in the areas of content, process, and product</p> <p>Misconceptions are generally addressed with the class as a whole</p> <p>The lesson design attempts to use inductive teaching but the examples presented only vaguely illustrate the targeted conceptual understandings</p> <p>A deductive design may also be included to support the learning of foundational facts and skills</p>	<p>Differentiation is based on an analysis of multiple data points that reveal individual student learning needs</p> <p>Specific accommodations are readily available based on anticipated student misconceptions and needs</p> <p>The lesson design is primarily inductive, requiring students to engage in a multifaceted inquiry process and to reflect on the connections across the examples presented so that students can formulate and defend their generalizations</p> <p>A deductive design may also be included to support the learning of foundational facts and skills</p>

SOURCE: Erickson and Lanning (2014, p. 62).

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FIGURE M8.3. THE DEVELOPING CONCEPT-BASED TEACHER: UNDERSTANDING CONCEPT-BASED CURRICULUM AND INSTRUCTION

	Novice		Emerging		Master	
<b>Support for Concept-Based Teaching and Learning</b>	<ul style="list-style-type: none"> <li>Names one or two reasons for concept-based teaching and learning</li> </ul>	<ul style="list-style-type: none"> <li>Articulates the major reasons for concept-based teaching and learning</li> </ul>	<ul style="list-style-type: none"> <li>Articulates the reasons for concept-based curriculum and instruction, citing relevant supporting research</li> </ul>			
<b>Components of Concept-Based Curriculum and Instruction</b>	<ul style="list-style-type: none"> <li>Concepts (macro-micro)</li> <li>Conceptual lenses</li> <li>Synergistic thinking</li> <li>Generalizations</li> <li>Guiding questions leveled and aligned with generalizations</li> <li>Critical knowledge and key skills</li> <li>Performance assessments versus activities</li> </ul>	<ul style="list-style-type: none"> <li>Can define some of the elements of a three-dimensional concept-based curriculum</li> <li>Model versus a two-dimensional model</li> <li>From a list of concepts, distinguishes macro- from micro-concepts</li> <li>Can define synergistic thinking but cannot yet explain methods to create synergistic thinking by students</li> </ul>	<ul style="list-style-type: none"> <li>Uses correct concept-based terminology but may not be clear about the rationale for each component</li> <li>When reviewing concept-based curriculum and/or instruction, recognizes the three-dimensional components</li> <li>Explains the difference between macro- and micro-concepts and states why it is important to know the difference</li> <li>Defines synergistic thinking and gives at least one example of an instructional technique/strategy to engage students' synergistic thinking</li> </ul>	<ul style="list-style-type: none"> <li>Explains the various components of concept-based curriculum using correct terminology and the rationale for each</li> <li>When reviewing a two-dimensional curriculum and/or lesson, suggests changes that will move the curriculum or lesson to a three-dimensional model</li> <li>Demonstrates a solid understanding of macro- and micro-concepts by explaining how to employ them accurately and effectively in concept-based curriculum and instruction</li> <li>Can explain the value of synergistic thinking and offers more than one personally created example of instructional techniques/strategies that clearly engage students' synergistic thinking</li> </ul>		
<b>Commitment to Continued Learning</b>	<ul style="list-style-type: none"> <li>Participates in concept-based presentations; attempts steps to adapt two-dimensional lessons to three-dimensional lessons</li> </ul>	<ul style="list-style-type: none"> <li>Participates in professional development presentations on concept-based curriculum and instruction and follows up by attempting to put new learning into practice</li> <li>Accepts formal or informal coaching and mentoring support</li> <li>Independently reads texts on concept-based curriculum and instruction</li> </ul>	<ul style="list-style-type: none"> <li>Participates in professional development presentations on concept-based curriculum and follows up by attempting to put new learning into practice</li> <li>Accepts formal or informal coaching and mentoring support</li> <li>Independently reads texts on concept-based curriculum and instruction</li> </ul>	<ul style="list-style-type: none"> <li>Passionately supports concept-based curriculum and instruction as evidenced by active participation in presentations or book study groups with follow-up implementation that includes ongoing communication and collaborative work with others (e.g., coaching and mentoring)</li> <li>Demonstrates continued effort to deepen understanding through sharing learning and leading professional development</li> <li>Continually reflects upon and refines practice</li> </ul>		

FIGURE M8.4: CONCEPT-BASED INSTRUCTION: TARGET SETTING

<p><b>Component of Concept-Based Instruction</b></p>	<p><b>Level (Novice, Emerging, or Master)</b> Why are you at this level?</p>	<p><b>Target Setting</b> How will you progress to the next level?</p>
<p><b>Lesson opening</b></p>		
<p><b>During a lesson</b> Instruction reflects modeling, facilitating, and mediating conceptual understanding</p>		
<p><b>Lesson closing</b></p>		

FIGURE M8.5: CONCEPT-BASED LESSON PLANNING: TARGET SETTING

<p><b>Component of Concept-Based Lesson Planning</b></p>	<p><b>Level (Novice, Emerging, or Master)</b> Why are you at this level?</p>	<p><b>Target Setting</b> How will you progress to the next level?</p>
<p><b>Lesson opening:</b> An explicit and engaging summary of the work to be accomplished that triggers synergistic thinking</p>		
<p><b>Learning targets:</b> What students are expected to know (factual knowledge), understand (generalization), and be able to do (skills)</p>		
<p><b>Guiding questions:</b> The three different types of questions (factual, conceptual, provocative) serve as a bridging tool for conceptual thinking and problem solving.</p>		
<p><b>Learning experiences:</b> Intellectually engaging student work that provide opportunities for students to practice their learning and to arrive at the target generalization (conceptual understanding)</p>		

<b>Component of Concept-Based Lesson Planning</b> <b>Assessment methods:</b> Assessment types are selected according to the lesson's learning targets (know, understand, do) and the assessment purposes (formative & summative) in order to capture evidence of students' learning (process & product), which will then inform instruction.	<b>Level (Novice, Emerging, or Master)</b> Why are you at this level?	<b>Target Setting</b> How will you progress to the next level?
<b>Differentiation:</b> Lesson adjustments are planned, as needed, for the content students are expected to master, the process students will use to access the content, and the product students will produce to show their learning. The conceptual understanding (generalization) all students are expected to realize remains consistent for all students.		

(Continued)



FIGURE 8.3: (CONTINUED)

<b>Component of Concept-Based Lesson Planning</b>	<b>Level (Novice, Emerging, or Master)</b> Why are you at this level?	<b>Target Setting</b> How will you progress to the next level?
<p><b>Lesson design:</b> In a deductive lesson design, the teacher states the learning targets (including generalization) to the learners at the beginning of the instruction. In an inductive design, students construct their understandings through an inquiry process.</p> <p><b>Closing:</b> Plan for a way that evidence of learning can be reviewed collectively.</p>		

FIGURE M8.6: UNDERSTANDING CONCEPT-BASED CURRICULUM AND INSTRUCTION: TARGET SETTING

<p><b>Component of Understanding Concept-Based Curriculum and Instruction</b></p> <p><b>Support for concept-based teaching and learning</b></p> <th data-bbox="371 252 718 1380"> <p><b>Level (Novice, Emerging, or Master)</b></p> <p>Why are you at this level?</p> <th data-bbox="718 252 1413 1380"> <p><b>Target Setting</b></p> <p>How will you progress to the next level?</p> </th></th>	<p><b>Level (Novice, Emerging, or Master)</b></p> <p>Why are you at this level?</p> <th data-bbox="718 252 1413 1380"> <p><b>Target Setting</b></p> <p>How will you progress to the next level?</p> </th>	<p><b>Target Setting</b></p> <p>How will you progress to the next level?</p>
<p><b>Components of concept-based:</b></p> <ul style="list-style-type: none"> <li><input type="checkbox"/> Concepts (macro-micro)</li> <li><input type="checkbox"/> Conceptual lenses</li> <li><input type="checkbox"/> Synergistic thinking</li> <li><input type="checkbox"/> Generalizations</li> <li><input type="checkbox"/> Guiding questions leveled &amp; aligned with generalizations</li> <li><input type="checkbox"/> Critical knowledge &amp; key skills</li> <li><input type="checkbox"/> Performance assessments versus activities</li> </ul> <p><b>Commitment to continued learning</b></p>		

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FIGURE M8.7: TEMPLATE FOR PERSONAL ACTION PLAN

## Personal Action Plan

**List your top five skills (things that you do well):**

- 1.
- 2.
- 3.
- 4.
- 5.

**List your top five strengths (could be a knowledge area):**

- 1.
- 2.
- 3.
- 4.
- 5.

**List three things you want to develop in terms of your teaching (list in order of priority):**

- 1.
- 2.
- 3.

**When and how will you achieve these three goals?**

FIGURE M8.8: TEMPLATE FOR SCHOOL OR FACULTY DEVELOPMENT PLAN USING SMART GOALS

SCHOOL/FACULTY DEVELOPMENT PLAN FOR DEVELOPING CONCEPT-BASED TEACHERS					
Specific Target WWWWW What, Why, Who, Where, Which	Measurable How will I know when this is accomplished?	Achievable/ Assignable Who will do this?	Relevant/Realistic Is this relevant to our school development plan?	Time Frame What time frame should we set?	Achieved? Y/N
<p>What: unit webs and unit planners</p> <p>Why: to write concept-based units of work and craft generalizations for a unit</p> <p>Who: the whole faculty</p> <p>Where: middle school</p> <p>Which: Grade 8</p>	<p>When we have written unit planners for each of the six units in Grade 8</p>	<p>The Grade 8 teaching team</p>	<p>The school development plan has identified concept-based curriculum and instruction as a target, and the senior leadership team has allocated collaborative sessions to support this process.</p>	<ul style="list-style-type: none"> <li>- Collaboration once per week for 90 minutes</li> <li>- Three months</li> </ul>	

Adapted from Doran, G. T. (1981). There's a S.M.A.R.T. way to write management's goals and objectives. *Management Review*, 70(11), 35-36.

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## Discussion Questions for Module 8

1. How will you develop certain aspects of concept-based lesson planning and instruction?
2. How will you encourage your faculty to start this journey and lead the change toward concept-based curriculum?
3. How will you use your current resources, planners, and standards to design a concept-based curriculum?
4. Which rubrics will you use from this module and why?

## An Opportunity for Reflection

**Connect: How are the ideas connected to what you already knew?**

**Extend: What ideas extended your thinking?**

**Challenge: What is still challenging for you?**