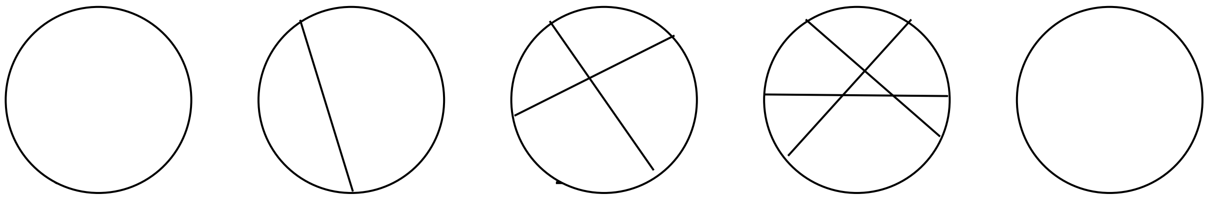
Figure 8.22. Cut the Cake

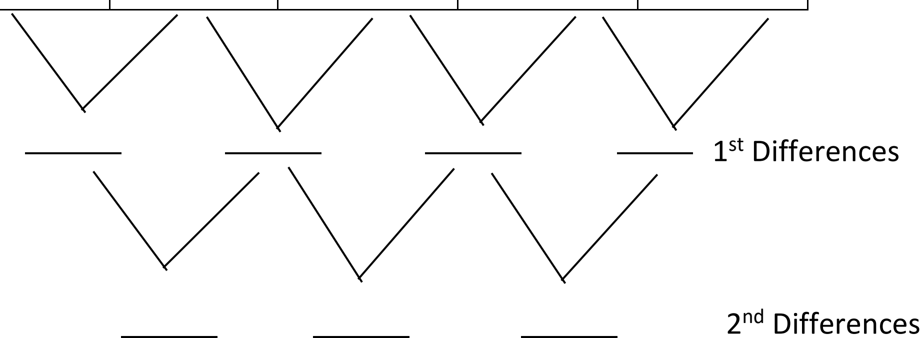
Kadi the crazy pastry chef decided that she wanted to cut her cakes into as many pieces with the fewest cuts as possible. She didn't even care that it would cause pieces that were irregular in size and shape. She began cutting cakes in the following manner:



Record the maximum number of pieces Kadi can get for each number of cuts.

Number of cuts	0	1	2	3	4
Number of pieces					

Kadi (being a math enthusiast) wanted to determine if her new way of cutting cakes was a polynomial function. She decided to see how the number of pieces was changing by looking at the differences.



Is the function linear? _____ How do you know?

Is the function quadratic? _____ How do you know?

What is the maximum number of pizza pieces you can obtain by making 5 cuts? ______
Explain how you figured this out.

Kadi thought this was way cool! She decided to play with quadratic functions some more! If the first numbers in each row are 1, 1, and 1 respectively, what is the quadratic relationship? (See hints below if needed!)

$$f(x) = ax^2 + bx + c$$

x	0	1	2	3	4
y = f(x)	1				

1st difference: ___1__ ______ ______ ______

2nd difference: __1___ ______ ______

Hints for steps:

1. The function is quadratic. What does that tell you about the 2nd difference?
2. Try to fill in the 1st differences.
3. Try to fill in the y-values or f(x) in the chart.
4. Think about the quadratic relation, $f(x) = ax^2 + bx + c$. If x = 0, what does y or f(x) equal? Where is this in the table?
5. If x = 1, what does y equal? If x = 2, what does y equal?
6. Can you use step 5 to find a system of equations to solve for a and b?

If the first numbers in each row are -2, 3, and 4 respectively, what is the quadratic relationship?

$$f(x) = ax^2 + bx + c$$

x	0	1	2	3	4
y = f(x)	-2				

1st difference: ___3__ _______ _______ _______

2nd difference: __4___ _______ _______

Kadi wondered why this worked, and if it could be made easier. She noticed the systems were the same both times except for the constants. She used the same process with only the algebraic symbols.

$$f(x) = ax^2 + bx + c$$

x	0	1	2	3	4
y = f(x)	c	$a + b + c$	$4a + 2b + c$		

1st difference: ___$a + b$__ _______ _______ _______

2nd difference: ______ _______ _______

Fill in the rest of the chart. What is always on the 2nd difference line? ________________

How could this be used to find the quadratic function equation given the first values in each row?

The ever-curious (but still crazy) Kadi questioned if this would work for cubic functions as well!

$$f(x) = ax^3 + bx^2 + cx + d$$

x	0	1	2	3	4	5
y = f(x)	13	17	27	55	113	213

1st difference: ______ ______ ______ ______ ______

2nd difference: ______ ______ ______ ______

3rd difference: ______ ______ ______

Find the cubic function for this table of values. If you like, do the next problem first.

Can you generalize for any cubic function?

$$f(x) = ax^3 + bx^2 + cx + d$$

x	0	1	2	3	4	5
y = f(x)						

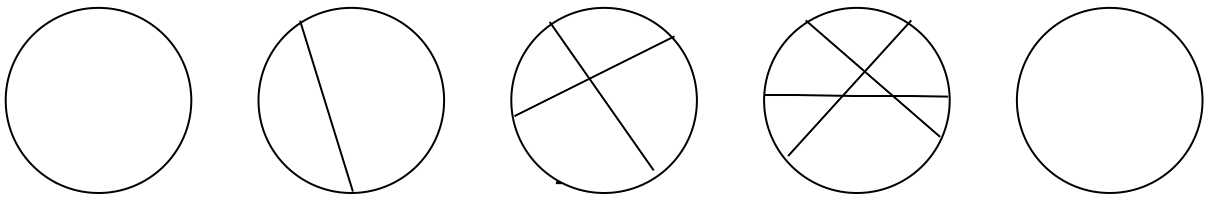
1st difference: ______ ______ ______ ______ ______

2nd difference: ______ ______ ______ ______

3rd difference: ______ ______ ______

Cut the Cake – Answer Key

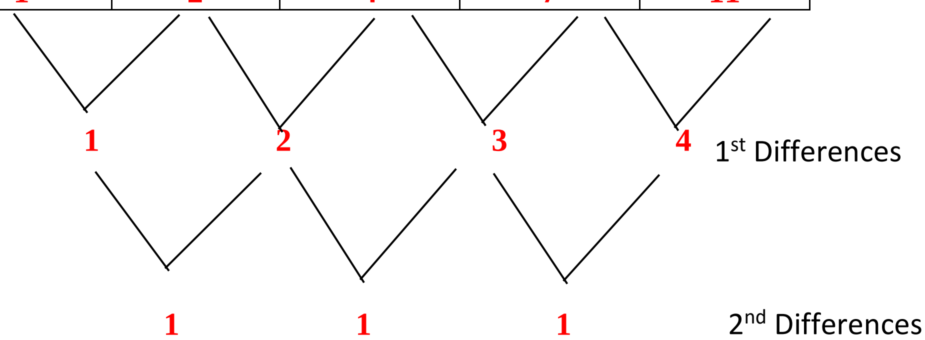
Kadi the crazy pastry chef decided that she wanted to cut her cakes into as many pieces with the fewest cuts as possible. She didn't even care that it would cause pieces that were irregular in size and shape. She began cutting cakes in the following manner:



Record the maximum number of pieces Kadi can get for each number of cuts.

Number of cuts	0	1	2	3	4
Number of pieces	**1**	**2**	**4**	**7**	**11**

Kadi (being a math enthusiast) wanted to determine if her new way of cutting cakes was a polynomial function. She decided to see how the number of pieces was changing by looking at the differences.



Is the function linear? **No** How do you know? **Not a constant rate of change on the first difference**

Is the function quadratic? **Yes** How do you know? **Constant rate of change on the second difference**

What is the maximum number of pizza pieces you can obtain by making 5 cuts? **16** Explain how you figured this out.

Answers vary

The first differences are increasing by 1 with each cut. So on the 5th cut, there would be (4+1) more cuts than on the 4th cut. 11 + 5 = 16 cuts.

Since the second difference is always 1, you can add back up the table. 4+1 = 5 for the first difference, and 11 + 5 = 16 for the table value.

Kadi thought this was way cool! She decided to play with quadratic functions some more! If the first numbers in each row are 1, 1, and 1 respectively, what is the quadratic relationship? (See hints below if needed!)

$$f(x) = ax^2 + bx + c$$

x	0	1	2	3	4
y = f(x)	1	**2**	**4**	**7**	**11**

1st difference: __1__ **2** **3** **4**

2nd difference: __1__ **1** **1**

Hints for steps:

1. The function is quadratic. What does that tell you about the 2nd difference?
2. Try to fill in the 1st differences.
3. Try to fill in the y-values in the chart.
4. Think about the quadratic relation, $f(x) = ax^2 + bx + c$. If x = 0. what does y equal? Where is this in the table?

5. If x = 1, what does y equal? If x = 2, what does y equal?
   (1, 2) **(2, 4)**

6. Can you use step 5 to find a system of equations to solve for a and b?

(0, 1) so 1 = *c*
(1, 2) so 2 = *a* + *b* + 1; *a* + *b* = 1
(2, 4) so 4 = 4*a* + 2*b* + 1; 4*a* + 2*b* = 3

Solve the system (or use substitution)

4*a* + 2*b* = 3
<u>*a* + *b* = 1</u>

4*a* + 2*b* = 3
<u>–2*a* – 2*b* = –2</u>
2*a* = 1
***a* = ½**

½ + *b* = 1
***b* = ½**

f(x) = 1/2x^2 + 1/2 *x* + 1

If the first numbers in each row are -2, 3, and 4 respectively, what is the quadratic relationship?

$$f(x) = ax^2 + bx + c$$

x	0	1	2	3	4
y = f(x)	-2	**1**	**8**	**19**	**34**

1st difference: ___3___ **7** **11** **15**

2nd difference: ___4___ **4** **4**

$c = -2$
$1 = a + b - 2$; $a + b = 3$ **$a + b = 3$**
$8 = 4a + 2b - 2$; $4a + 2b = 10$ **$\underline{4a + 2b = 10}$**

(solving the system) a = 2, b = 1, c = –2
$f(x) = 2x^2 + x - 2$

Kadi wondered why this worked, and if it could be made easier. She noticed the systems were the same both times except for the constants. She used the same process with only the algebraic symbols.

$$f(x) = ax^2 + bx + c$$

x	0	1	2	3	4
y	c	$a + b + c$	$4a + 2b + c$	**$9a+3b+c$**	**$16a+4b+c$**

1st difference: ___$a + b$___ **$3a + b$** **$5a + b$** **$7a + b$**

2nd difference: **$2a$** **$2a$** **$2a$**

Fill in the rest of the chart. What is always on the 2nd difference line? **$2a$**

How could this be used to find the quadratic function equation given the first values in each row?
The second difference divided by 2 = *a* in the quadratic function. *b* = the first value of first differences minus *a*. And *c* = the first table value when *x* = 0.
The ever-curious (but still crazy) Kadi questioned if this would work for cubic functions as well!

$$f(x) = ax^3 + bx^2 + cx + d$$

x	0	1	2	3	4	5
y	13	17	27	55	113	213

1st difference: **4** **10** **28** **58** **100**

2nd difference: **6** **18** **30** **42**

3rd difference: **12** **12** **12**

Find the cubic function for this table of values. If you like, do the next problem first.
Can be solved using a 3X3 system, or wait until pattern is discovered below.

6*a* = 12; *a* = 2
6*a* +2*b* = 6; *b* = –3
***a* + *b* + *c* = 4; c = 5**
***d* = 13**
f(x) = $2x^3 - 3x^2 + 5x + 13$

Can you generalize for any cubic function?

$$f(x) = ax^3 + bx^2 + cx + d$$

x	0	1	2	3	4	5
y	***d***	***a*+*b*+*c*+*d***	**8*a*+4*b*+2*c*+*d***	**27*a*+9*b*+3*c*+*d***	**64*a*+16*b*+4*c*+*d***	**125*a*+25*b*+5*c*+*d***

1st difference: ***a* + *b* + *c*** **7*a* + 3*b*+ *c*** **19*a* + 5*b*+ *c*** **37*a* + 7*b*+ *c*** **61*a* + 9*b*+ *c***

2nd difference: **6*a* + 2*b*** **12*a* + 2*b*** **18*a* + 2*b*** **24*a* + 2*b***

3rd difference: **6*a*** **6*a*** **6*a***