Figure 8.17. Beyond Linear: Working with Polynomials Unit Design Template

Unit Title: Beyond Linear: Working with Polynomials (Algebra 2)

Standards Addressed (from Common Core State Standards, NGO and CCSSO, 2010):

Polynomial Arithmetic:

- Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- Rewrite simple rational expressions in different forms; write $^{a(x)}/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$,where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
- Know and apply the Remainder Theorem: For a polynomial *p(x)* and a number *a*, the remainder on division by $x – a$ is *p(a)*, so *p(a)* = 0 if and only if *(x – a)* is a factor of *p(x)*.

Polynomial Equations:

- Know there is a complex number *i* such that *i2* = –1, and every complex number has the form *a + bi* with *a* and *b* real. *(Review from last unit)*
- Use the relation *i2* = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. *(Review from last unit)*
- Solve quadratic equations with real coefficients that have complex solutions. *(Review from last unit)*
- Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).*
- Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity (x2 + y2)2 = (x2 – y2)2 + (2xy)2 can be used to generate Pythagorean triples*

Polynomials Functions:

- Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior.
- Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- Graph functions expressed symbolically and show key features of the graph (by hand in simple cases and using technology for more complicated cases).

By the end of the unit (new learning), what will students …

Know	Understand	Be Able to Do
Vocabulary: binomial expansion, complex conjugates, complex numbers, continuous functions, degree, end behaviors, even function, factor by grouping, intervals, Fundamental Theorem of Algebra, imaginary numbers, odd functions, Pascal's triangle, polynomial and polynomial functions, relative (local) maximums and minimums, Rational Root theorem, Remainder Theorem, repeated differences (finite differences), repeated solution, symmetry, synthetic division, Determining degree of polynomial, and polynomial equation from a table of values. Strategies for operations with polynomials (e.g. lattice multiplication and division; long division; synthetic division) Remainder Theorem Rational Root Theorem Fundamental Theorem of Algebra Binomial expansion using Pascal's triangle How to identify key component of a polynomial	Polynomials are very similar to integers. Arithmetic with polynomials works in the same ways as arithmetic with integers. They are closed in addition, subtraction, and multiplication, just as are integers. (Algebra is grown up arithmetic.) Polynomial functions follow all of the same general patterns as any other functions with parent functions, transformations, domain and rage, representations, etc. Polynomial functions are used to model, analyze and make predictions in many real world situations. Defining an imaginary number, *i*, explains many mathematical anomalies including polynomial functions without a Real zero, and defines roots for polynomial functions whose graphs do not cross the x-axis.	Identify polynomials and polynomial functions. Operate on polynomials using multiple strategies. Rewrite simple rational expressions using inspection and long division. Use Pascal's triangle to expand binomials. Explain how operating on polynomials is like operating on integers. Apply the factor and remainder theorems to factor or evaluate polynomials. Prove polynomial identities and find Pythagorean Triples. Graph polynomials and identify key features. Factor polynomials using multiple methods. Compare and contrast strategies used with polynomials (operations, expansion, factoring, evaluating, etc.) Find all zeros of a polynomial function, including imaginary zeros.

graph Analyzing a polynomial to sketch its graph $i^2 = -1$ Cycles of imaginary numbers A complex number is composed of an imaginary number and a real number. Not all roots of a function are real. Some roots of a function can occur multiple times. How to determine rational roots for a polynomial function		Explain how a polynomial function has the same number of roots as its degree even though some graphs appear to have less or no roots. Apply polynomials in real world contexts in order to analyze, solve or make predictions concerning the situation.

Pre-Assessment Ideas:
Quick Write: What do you know about Polynomials? (Provide word bank with all vocabulary in unit) Fill in categories:
Operations (Give problems to complete, ask for multiple strategies)
Representations
Finding zeros
Analyzing a graph

Review: Complex number review problems

Summative Assessment Ideas:
Chapter test
Authentic Performance Assessment: Choose an application for Polynomial modeling: Roller Coasters, Engineering Packaging, Research various tables of data that are modeled by a polynomial – Scatter plot, fit a polynomial model and interpret the results, Create a Game to practice operations and graphing polynomials, Write a "Do-It-Yourself Polynomials" book

Formative Assessment Ideas:
Exit cards, homework, class activities, discourse, quiz and quest