Additional Understandings in Secondary Mathematics

The following tables provide additional Understanding statements for many of the topics and skills in middle and high school mathematics. This in no way is to be interpreted as a complete list or curriculum, but rather supplemental information for teachers endeavoring to unpack standards and build concept-based units.

Middle School

Grade	Topic	Skill	Understanding
6	Number Sense	• Order and place rational numbers on number line • Analyze proportional relationships • Introduction to Integers	• Our numbers follow a pattern that remains the same. • Our numbers are based on groups of ten. • A negative sign in mathematics means opposite. • The sign of a number represents different contexts in our world. • Our numbers can be represented in many equivalent forms.
6	Operations	• Fluently divide multi-digit numbers using the standard algorithm • Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation • Interpret and compute quotients of fractions and solve word problems involving division of fractions by	• There are many strategies that can be used to perform any operation, but it is helpful to have one method that we agree to call standard. • The meaning of an operation determines which operation to use in a given situation. • Only things that are alike can be added or subtracted. • Multiplication can represent repeated addition, a number of groups with equal elements, and the creation of area. • Division can represent repeated subtraction,

		fractions	putting a total number of items into equal groups, and the partitioning.
6	Algebra	- Develop understanding of variables - Write, read and evaluate algebraic expressions - Write and solve one-variable equations and inequalities - Graph points in four quadrants	- Algebra is sophisticated arithmetic. - A variable can represent an unknown value, or a range of unknown values. - The same quantity or expression can be represented in many equivalent ways. - We use algebra to model the real world. - Operations used with algebraic symbols follow the same patterns and meanings as using the operation in any number system. - Only things that are alike can be added or subtracted. - Our operations come in inverse pairs, which allow us to solve algebraic equations. - A negative sign in mathematics means opposite. - Number lines can be used to represent and graph regions or space.
6	Geometry	- Area, Volume and Surface Area - Apply Algebra to Geometry including graphing in coordinate plane and solving volume formulas	- Measurement lets us describe and compare objects according to their attributes and properties. - What we measure determines the units used to measure, and the units of measure describe the attribute being measured.

			• We use algebra to model the real world. • Number operations are often used to determine the measured attributes of a shape.
6	Statistics	• Represent, describe and interpret numerical data in a variety of ways • Interpret numerical data through measures of center, spread, and shape of distribution	• Mean median and mode all represent what is typical or "average" however, the best representation will depend on the situation and type of data. • The type of graph is influenced by what data is being collected. • Data is compared and analyzed with different statistical calculations.
7	Number	• Analyze proportional relationships and solve proportional situations • Represent rational numbers in multiple formats • Apply properties of operations to add, subtract, multiply and divide rational numbers	• Ratios and proportions describe a predictable multiplicative relationship between quantities that can change. • Proportions describe real world relationships and allow us to make comparisons. • Our numbers can be represented in many equivalent forms. • A negative sign in mathematics means opposite. • Only things that are alike can be added or subtracted. • Rational numbers can be used to represent situations in the real world which include both positive and negative values. • Operations with rational numbers follow

			the same patterns as operations with integers.
7	Algebra	- Apply properties of operations to simplify or generate equivalent expressions - Construct and solve algebraic expressions, equations and inequalities to represent mathematical problems (including real-world).	- Algebra is sophisticated arithmetic. - A variable can represent an unknown value, or a range of unknown values. - The same quantity or expression can be represented in many equivalent ways. - We use algebra to model the real world. - Operations used with algebraic symbols follow the same patterns and meanings as using the operation in any number system. - Only things that are alike can be added or subtracted. - Our operations come in inverse pairs, which allow us to solve algebraic equations. - A negative sign in mathematics means opposite.
7	Geometry	- Analyze geometric shapes and describe relationships among them - Solve contextual problems involving angle measure, area, surface area and volume of .	- Three-dimensional shapes are built from two-dimensional shapes. - The characteristics of angles and definitions of angle relationships allow you to set up and solve problems algebraically. - Specific conditions must be met to form unique triangles, one triangle, or no triangle. - Areas cover, surface

			areas wrap, and volumes fill.
7	Statistics and Probability	• Draw inferences about populations, including comparing two populations • Develop and use probability models including probability of compound events	• Statistics can be used to gain information about a population by examining a sample of the population. • Generalizations about a population from a sample are valid only if the sample is representative of that population. • Probability is the likelihood of something happening, but it doesn't guarantee results. • There are different ways to organize outcomes in order to determine probabilities. • Experimental probability may not be the same as theoretical, but the more repetitions you do, the closer it should get.
8	Number	• Expand the Real Number System by working with irrational numbers. • Understand the values and rules with expressions involving integer exponents including exponent "rules" and scientific notation	• Our numbers can be represented in many equivalent forms. • Every number can be written as a decimal expansion, but some can't be written exactly. • The operation of "squaring" is the same as finding the area of a square and "cubing" is finding the volume of a cube. Likewise, a square root finds the length of a side of a square of a given area and a cube root finds the edge of a cube with

			a given volume.
			• Only things that are alike can be added or subtracted.
			• Different forms of numbers show different aspects of numbers, and can be used for comparisons.
			• Exponent rules come from properties of multiplication and division.
8	Algebra	• Analyze and solve linear equations and systems of linear equations	• Algebra is sophisticated arithmetic.
			• Only things that are alike can be added or subtracted.
			• Algebraic equations model and solve real world situations.
			• Variables represent unknown, given, or varying values.
			• In equations, the equal sign shows an equivalence of two quantities that must be maintained.
8	Functions	• Define, evaluate and compare linear functions • Describe functional relationships from a graph qualitatively	• A function determines a specific relationship between co-varying variables.
			• All function families have distinct characteristics.
			• Linear functions are determined by their rate of change.
			• Functions can be represented in several different ways (algebraically/symbolically, graphically, numerically/ tables or

			words). Each with different strengths and weaknesses.
8	Geometry	• Define, describe and apply similarity and congruence of shapes • Explain and apply the Pythagorean Theorem • Solve contextual problems involving volume of cones and spheres	• Congruence and similarity provide specific information as to how shapes compare to one another, which can then be used to find missing information and • All transformations preserve similarity but some also preserve congruence. • Definitions and theorems about angle relationships allow us to prove relationships and solve problems algebraically. • A right triangle has a special unique relationship among the sides and angles. • Three dimensional shapes are built from two-dimensional shapes. • The types of unit reflect what is being measured, and what is being measured determines the types of unit.
8	Statistics and Probability	• Investigate and describe representations of bivariate data • Use the equation of a line of best fit to interpret data in context	• Data can be represented and analyzed in different ways. • Two different sets of data can be related and compared. • Patterns in visual representations of data show important information about what is being:

			researched analyzed questioned discovered • Functions can be used to model and analyze data.

High School

Note: Algebra I and Algebra II courses are grouped together as "Algebra" because the Understandings should remain consistent although the skills and specific topics progress.

Algebra	Number System	• Properties of and working with rational exponents • Properties of and working with rational and irrational numbers • Properties of and working with complex numbers	• Number systems can be based on definition or man-made. • Our number system classifies and categorizes numbers based on their characteristics. • Operations with rational exponents follow the same principles as integer exponents.
Algebra	Algebraic Arithmetic Note: Included in this section are all algebraic structures including polynomial, roots, exponents, logarithms, etc.	• Use algebraic structure to interpret expressions, write equivalent expressions, and solve problems. • Write expressions, equations and inequalities to model real world contexts, and solve real world problems, including systems of equations. • Apply arithmetic operations on polynomials.	• Algebra is grown up arithmetic. • Only things that are alike can be added or subtracted. • A negative in mathematics means "opposite." • Operations work in inverse pairs and in specific order in the solution process. • Numbers and quantities can be expressed in many equivalent ways, and a different form may be more effective in analyzing and solving a problem. (Factors, squaring,

		• Find zeros of factors of polynomials, and relate these to the roots of a function and its graph.	equivalent fractions) • The relationship of equality must be maintained throughout the solution process. • Systems of equations and inequalities model real-life situations to solve problems, make comparisons and predict what may happen in the future or what has happened in the past. • There are many different methods to solve equations or systems of equations, each with strengths and weaknesses given the structures and contexts of the equations. • Inequalities build on the same principals and properties as equations. • Not all solutions are a single answer, and not all answers are solutions. • Polynomials form a system analogous to the integers
Algebra	Functions Note: Included in the term "function" in the skills section are all types of functions including linear, quadratic and other polynomial functions, root functions,	• Understand and identify functions in all representations. • Represent, analyze and interpret functions in multiple representations. • Construct functions from other functions	• A function is a specific kind of relationship between two variables and is effective in modeling our world. • Functions can be represented in different ways (words, tables, graphs, symbols), and each form provides different aspects of the function more effectively. From any

	exponential, logarithmic, , etc.	using operations and inverses. • Use functions to model and solve real world context problems.	one form the others can be found. • Families of functions share the same characteristics and properties in all representations. • Descriptions of how a function changes, and its pattern of rate of change, identify the type of function and can be quantified. • Functions can be built from other functions by use of operations, composition and inverses. • Not all values of the domain can be used in every function. The allowable values of the domain determine the values of the range. • Sequences and series are functions, and can also describe the patterns within functions.
Algebra and/or Geometry	Trigonometry	• Define and use radian measure of an angle; explain the relationship and convert between radians and degrees. • Show and use the relationships among the unit circle, similar triangles and trigonometric functions (including Pythagorean	• There are different measurement systems that can be equated, but most have more common uses in specific contexts and areas of study. • The unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers. • Trigonometric functions follow the same patterns and operations as all other

		identities) • Define trigonometric ratios and use them to solve problems. • Model, interpret and solve periodic contexts with trigonometric functions	functions. • Trigonometry can be used to model cyclical situations and allow for calculations of indirect measurement in the real world.
Geometry	Geometry	• Define, represent, and complete transformations in the plane, including representing transformations as functions. • Define, justify and prove congruence and similarity through transformations. • Prove theorems about lines, angles, triangles, quadrilaterals, and circles. • Use algebra to describe transformations, write equations of conic sections, and prove geometric theorems and shapes in the coordinate plane. • Apply geometric formulae and principles to real world contexts.	• Geometry defines and analyzes space and shape in our world. • Points, lines, and planes are the building blocks of all space (2-D and 3-D). • Polygons and polyhedra are defined, classified and compared based on their properties and characteristics. • Geometric shapes, transformations and applications can be represented in multiple ways, and can be analyzed algebraically. • Coordinate space provides the framework for numerical analysis of geometric figures. • Precise definitions and properties come together to form mathematical proof. • Similarity and congruence are based on the characteristics preserved through transformations, which in turn provides the basis for proof by theorems as well.

			• Specific relationships exist among sides and angles in right triangles that allow for calculations of indirect measurement. • The geometric aspect being measured determines the type of unit used based on the dimensions being multiplied, and the type of unit of measurement describes what is being measured.
Probability & Statistics	Statistics	• Summarize, represent and interpret data with one or two variables. • Select appropriate models for data, and interpret critical features. • Draw inferences and justify or refute conclusions in statistical experiments.	• Statistics is a process for drawing conclusions about a population based on a random sample from that population. • Data can be organized and represented in various ways for analysis, to show trends and to draw conclusions to predict future events. • There are various distributions that model and help to analyze different situations and data. • Different sampling techniques apply to different situations.
Probability & Statistics	Probability	• Understand, distinguish between and model independent and conditional events. • Construct and	• Probability describes the likelihood of an event occurring and can be used to make predictions and decisions. • The more experiments you conduct the closer

		interpret representations of data and models of probability for the same data. • Use the rules of probability to compute the probability of compound events.	you get to a theoretical result. • Data can be organized and represented in various ways for analysis, to show trends and to draw conclusions to predict future events.